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Comparative Analysis of Conventional and Neutron-Skin-Modified Models for Cluster Radioactivity and Spontaneous Fission in Superheavy Nuclei

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ABSTRACT

Neutron skin thickness is an important structural property of neutron-rich superheavy nuclei because variations in the spatial extension of the neutron distribution can modify nuclear geometry, surface properties, interaction barriers, and radioactive-decay behaviour. This study presents a systematic comparative re-analysis of two previously developed approaches for cluster radioactivity and spontaneous fission: the conventional density-dependent cluster model within the Wentzel–Kramers–Brillouin (DDCM–WKB) framework and a unified neutron-skin-modified formulation that explicitly incorporates neutron skin thickness into nuclear geometry and barrier-dependent decay quantities. The objective is to determine quantitatively whether explicit neutron-skin treatment improves predictive performance relative to the conventional formulation. Cluster-radioactivity calculations exhibit pronounced nucleus-dependent sensitivity to barrier penetrability, Q-value, shell structure, daughter-nucleus configuration, and neutron skin effects, whereas spontaneous fission shows a more systematic variation with neutron skin thickness within the modified formulation. The conventional and neutron-skin-modified formulations previously reported root-mean-square (RMS) deviations of 1.822 and 1.437, respectively, for the available experimental spontaneous-fission half-life data. Direct comparison of these values in the present study corresponds to an RMS reduction of approximately 21.1%.

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Nucleus-resolved analysis further shows that the neutron-skin-modified formulation provides the closer prediction for six of the nine nuclei in the common experimental validation subset and substantially reduces systematic underprediction. However, the improvement is not universal across individual nuclei. Overall, the results indicate that explicit neutron-skin treatment improves the description of the available spontaneous-fission half-life data while neutron skin thickness remains one of several structural factors governing the competition between cluster radioactivity and spontaneous fission.

Keywords: neutron-skin thickness, superheavy nuclei, cluster radioactivity, spontaneous fission, density-dependent cluster model, WKB approximation, fission barrier, decay half-life.

1. INTRODUCTION

The stability of superheavy nuclei (SHN) remains a major problem in nuclear physics because the existence of nuclei with very large proton numbers depends on a delicate competition between the attractive nuclear interaction and the increasingly strong Coulomb repulsion. Microscopic shell effects can provide additional stabilization and support the predicted island of stability; however, the synthesis and identification of SHN remain experimentally challenging because of their extremely small formation cross sections and generally short radioactive half-lives [1, 2]. Reliable theoretical descriptions of their decay mechanisms are therefore essential for understanding their stability and for identifying nuclei and decay pathways of potential experimental interest.

Cluster radioactivity and spontaneous fission are among the decay processes that provide important information on the structure and stability of heavy and superheavy nuclei. Cluster decay involves the quantum-mechanical emission of a nuclear fragment and is governed principally by the decay energy, cluster preformation probability, interaction potential, barrier penetrability, shell structure, and daughter-nucleus configuration [3–5]. Because cluster emission occurs through quantum tunnelling, its theoretical treatment is commonly formulated within the Wentzel–Kramers–Brillouin (WKB) approximation [6, 7]. Spontaneous fission similarly involves tunnelling through a deformation-dependent barrier, but its lifetime is particularly sensitive to nuclear surface and Coulomb energies, shell corrections, pairing correlations, deformation, and collective dynamics [8–10]. The relative half-lives of these decay modes consequently provide useful information about the mechanisms controlling the survival of superheavy nuclei.

Considerable progress has been made in predicting these decay processes. Density-dependent cluster models, unified decay formulas, microscopic nuclear-density calculations, and phenomenological approaches have improved predictions of cluster-radioactivity properties in heavy and superheavy nuclei [11–14]. Likewise, microscopic and semi-empirical formulations incorporating shell effects, deformation, pairing correlations, collective inertia, and fission-barrier properties have advanced the prediction of spontaneous-fission half-lives [9, 10, 15, 16]. Despite these developments, the extreme neutron excess of many SHN introduces an additional structural quantity whose influence on competing decay mechanisms requires further investigation: neutron skin thickness.

Neutron skin thickness is defined as the difference between the neutron and proton root-mean-square radii and represents the spatial extension of the neutron distribution beyond the proton distribution. It is closely related to isospin asymmetry and to the density dependence of the nuclear symmetry energy [17–19].

Variations in neutron skin thickness can modify the effective nuclear radius, surface properties, density distribution, and interaction geometry and may therefore affect barrier-dependent nuclear processes [20–22]. Nevertheless, conventional radioactive-decay models generally account for neutron-rich effects indirectly through isospin asymmetry, microscopic densities, phenomenological radii, or effective interaction parameters rather than introducing neutron skin thickness explicitly into the quantities governing barrier formation and tunnelling.

The relationship between neutron skin thickness and competing decay modes in superheavy nuclei was investigated previously using a conventional density-dependent cluster model within the WKB framework [23]. In that treatment, cluster-decay penetrabilities and half-lives, and spontaneous-fission barrier characteristics and half-lives were calculated and subsequently examined in relation to neutron skin thickness. The results indicated that the competition between cluster radioactivity and spontaneous fission evolves across the investigated superheavy nuclei and that cluster decay is strongly influenced by Q-value, barrier penetrability, shell structure, and daughter-nucleus configuration [23]. Neutron skin thickness in this approach therefore served principally as a structural quantity against which the calculated decay properties could be examined rather than as an explicit modification of the interaction potential and tunnelling barrier.

A subsequent formulation treated neutron skin thickness explicitly within the decay model [24]. The unified neutron-skin-modified approach incorporated neutron skin thickness into the nuclear radius, surface separation, surface-energy contribution, interaction potential, barrier characteristics, tunnelling probability, decay constant, and half-life calculations for cluster radioactivity and spontaneous fission [24]. The physical distinction between the two formulations is therefore fundamental: the conventional approach evaluates the systematic relationship between neutron skin thickness and decay behaviour, whereas the modified approach allows neutron skin thickness to participate directly in determining the geometry and energetics of the decay barrier. The conventional DDCM–WKB and neutron-skin-modified formulations have previously reported RMS deviations of 1.822 [23] and 1.437 [24], respectively, for the available experimental spontaneous-fission half-life data, indicating lower aggregate prediction error for the modified formulation.

The difference in overall RMS deviation, however, does not by itself establish that the neutron-skin-modified formulation performs uniformly better for individual nuclei or explain where the improvement originates. An aggregate error measure can conceal nucleus-dependent differences, including cases in which the conventional formulation may provide the closer prediction. Moreover, comparison of overall RMS values alone cannot determine whether improvements are systematically associated with neutron skin thickness or whether they depend on other structural quantities governing cluster emission and spontaneous fission. A nucleus-by-nucleus comparative assessment is therefore required to establish the extent, consistency, and physical significance of the differences between the two formulations.

The present study addresses this problem through a systematic comparative re-analysis of the conventional DDCM–WKB approach [23] and the unified neutron-skin-modified formulation [24]. Both approaches are evaluated on a common basis using their calculated decay observables and the available experimental spontaneous-fission half-life data. In addition to comparing cluster-decay penetrability, decay constant and half-life, and spontaneous-fission barrier height, probability and half-life, the analysis examines nucleus-specific deviations from experiment, absolute logarithmic errors, signed errors, and aggregate measures of predictive performance. Particular attention is given to identifying nuclei for which explicit neutron-skin treatment improves agreement with experiment and those for which the conventional formulation remains more accurate.

Accordingly, the purpose of the present work is not to redevelop the two previously published formulations [23, 24], but to determine quantitatively what is gained and what limitations remain when neutron skin thickness is changed from a correlated structural indicator to an explicit component of the decay formalism. This comparative framework separates the previously established model formulations from the new model-to-model assessment and permits the predictive improvement to be evaluated at both aggregate and individual-nucleus levels. The analysis thereby provides a more rigorous basis for assessing the role of neutron-rich surface structure in the competition between cluster radioactivity and spontaneous fission in superheavy nuclei.

2. MATERIALS AND METHODS

2.1. Comparative Study Design

The present study constitutes a comparative re-analysis of two previously published theoretical treatments of cluster radioactivity and spontaneous fission in superheavy nuclei: the conventional density-dependent cluster model within the Wentzel–Kramers–Brillouin (DDCM–WKB) framework [23] and the unified neutron-skin-modified formulation [24]. The theoretical derivations of the two approaches have been reported previously and are not presented here as new model developments. Instead, their calculated decay observables are evaluated within a common comparative framework to determine quantitatively the consequences of treating neutron skin thickness explicitly rather than examining it principally as a structural quantity associated with decay systematics.

The comparison considers neutron skin thickness, decay Q-value, cluster-decay barrier penetrability, decay constant and half-life, together with spontaneous-fission barrier height, decay constant and half-life. Nuclear mass information used in the underlying calculations was obtained from AME2020 [25], whereas the available experimental spontaneous-fission half-lives used for validation were obtained from NUBASE2020 [26]. The source calculations confirm that mass excesses were taken from AME2020 for determination of the decay Q-values.

The comparative procedure was conducted at two levels. First, differences in the predicted decay observables were examined across the investigated neutron-skin range. Second, theoretical spontaneous-fission half-lives were compared nucleus by nucleus with the available experimental values. The latter analysis extends the comparison of the previously reported RMS values by introducing nucleus-specific residual and absolute-error measures, thereby determining whether the effect of explicit neutron-skin treatment is systematic or dependent on individual nuclear configurations. The original contribution of the present study is therefore the common nucleus-resolved comparative evaluation of the two previously published formulations, rather than the underlying model derivations or source calculations themselves.

2.2. Conventional DDCM–WKB Approach

In the conventional approach [23], cluster emission is described as a quantum-tunnelling process. The decay constant is determined by the cluster preformation probability, assault frequency, and barrier penetrability as

$$\lambda_c = S_c \nu P, \tag{1}$$

where S_c is the cluster preformation probability, ν is the assault frequency, and P is the barrier penetrability.

Within the WKB approximation, the barrier penetrability is expressed as

$$P = exp\left[-\frac{2}{\hbar}\int_{r_{in}}^{r_{out}}\sqrt{2\mu[V(r)-Q]}dr\right], \quad (2)$$

where μ is the reduced mass of the cluster–daughter system, $V(r)$ is the interaction potential, Q is the decay energy, and r_{in} and r_{out} are the inner and outer classical turning points, respectively, defined by $V(r_{in}) = V(r_{out}) = Q$. The corresponding cluster-decay half-life is

$$T_{1/2}^{c} = \frac{\ln 2}{\lambda_c}\,. \quad (3)$$

These quantities correspond to the conventional WKB treatment used to calculate cluster penetrability, decay constant, and half-life in the underlying study.

For spontaneous fission, the fission-barrier height is determined from the difference between the potential energies at the saddle-point and ground-state configurations as [27]

$$B_f = V(\beta_s) - V(\beta_g), \quad (4)$$

where β_s and β_g are the deformation parameters corresponding to the saddle-point and ground-state configurations, respectively.

The spontaneous-fission decay constant is expressed as

$$\lambda_{SF} = \nu_{SF}P_{SF}, \quad (5)$$

where ν_{SF} is the spontaneous-fission assault frequency and P_{SF} is the corresponding barrier penetrability.

The corresponding spontaneous-fission half-life is

$$T_{1/2}^{SF} = \frac{\ln 2}{\lambda_{SF}}. \quad (6)$$

The underlying conventional calculations employ this barrier-tunnelling framework to obtain spontaneous-fission barrier heights, decay probabilities and half-lives.

In this approach, neutron-rich structural effects enter through the conventional nuclear, isospin and barrier quantities. Neutron skin thickness is subsequently related to variations in the calculated decay observables rather than being introduced explicitly as an independent modification of the nuclear radius and interaction potential.

2.3. Unified Neutron-Skin-Modified Approach

The unified neutron-skin-modified formulation [24] retains the basic tunnelling framework while introducing neutron skin thickness explicitly into the nuclear geometry and barrier-dependent quantities. Neutron skin thickness is defined as

$$\Delta r_{np} = r_n - r_p, \quad (7)$$

where r_n and r_p are the neutron and proton root-mean-square radii, respectively.

In the modified formulation, Δr_{np} contributes explicitly to the modified matter radius defined in the underlying model [24]. The resulting geometrical correction propagates into the surface separation, reduced radius, surface-energy coefficient and proximity interaction [28].

The cluster-decay effective potential can therefore be represented schematically as

$$V_{eff}^{(skin)}(R) = V_C(R) + V_N^{(skin)}(R) + V_\ell(R), \quad (8)$$

where V_C , V_N and V_ℓ denote the Coulomb, neutron-skin-modified nuclear, and centrifugal contributions, respectively. In the underlying formulation, the neutron-skin correction enters the nuclear interaction through the modified matter radius, surface separation, reduced radius, and surface-energy coefficient [24].

The corresponding neutron-skin-modified WKB action integral is

$$K_{skin} = \frac{1}{\hbar c}\int_{R_2}^{R_3} \sqrt{2\mu c^2 \left[V_{eff}^{(skin)}(R) - Q\right]}\, dR, \quad (9)$$

where R_2 and R_3 are the inner and outer barrier turning points, respectively, μc^2 is the rest energy of the cluster-daughter system, $\hbar$ is the reduced Planck constant, and Q is the decay energy.

The corresponding neutron-skin-modified WKB penetrability is

$$P_{skin} = \exp(-2K_{skin}). \quad (10)$$

The associated neutron-skin-modified cluster-decay half-life is

$$T_{1/2}^{(skin)} = \frac{\ln 2}{S_c \nu_{skin} P_{skin}}. \quad (11)$$

where S_c is the cluster preformation factor, ν_{skin} is the assault frequency, and P_{skin} is the neutron-skin-modified WKB penetrability.

For spontaneous fission, neutron-skin effects are incorporated into the deformation-dependent potential through modifications of the surface, Coulomb, and symmetry-energy contributions. The corresponding neutron-skin-modified fission-barrier height is

$$B_f^{(skin)} = V_{SF}^{(skin)}(\, q_s \,) - V_{SF}^{(skin)}(q_g), \quad (12)$$

where $V_{SF}^{(skin)}$ is the neutron-skin-modified deformation-dependent fission potential, q_s denotes the saddle-point deformation, and q_g denotes the ground-state deformation.

The resulting barrier enters the neutron-skin-modified WKB action and consequently affects the penetrability, decay constant, and spontaneous-fission half-life. Thus, in contrast to the conventional formulation, neutron skin thickness participates explicitly in determining nuclear geometry, interaction-potential characteristics, and quantum tunnelling. Complete derivations, parameter values, and physical assumptions associated with the conventional and neutron-skin-modified formulations are those reported in [23] and [24], respectively. The equations presented here therefore establish the theoretical quantities required for the comparative analysis without reproducing the complete previously published derivations.

2.4. Nuclear and Experimental Datasets

The investigated dataset consists of the superheavy nuclei considered in the two underlying formulations. Nuclear mass excesses for the parent nuclei and corresponding decay products were obtained from AME2020 [25] and used to calculate the relevant Q-values. Neutron skin thickness was evaluated according to Eq. (7).

Across the investigated systems, the calculated neutron skin thickness increases from approximately $0.157\ fm$ for ^{260}Lr to $0.210\ fm$ for ^{322}Og.

Experimental validation was restricted to spontaneous-fission half-lives for nuclei for which corresponding experimental and calculated values were available. The experimental values were taken from NUBASE2020 [26] and the same validation subset was applied to both theoretical approaches. The underlying study explicitly identifies this dataset as the comparison between calculated spontaneous-fission half-lives and available NUBASE2020 experimental values.

No equivalent experimental validation of the calculated cluster-decay half-lives is claimed in the present comparative analysis. Cluster-decay results are instead compared between the two theoretical formulations to assess the effect of explicitly incorporating neutron skin thickness.

2.5. Quantitative Comparative Analysis

Because nuclear half-lives span many orders of magnitude, comparisons with experiment were conducted in base-10 logarithmic space. For nucleus i and model m, the signed logarithmic residual is defined as

$$E_{i,m} = \log_{10} T_{1/2,i}^{model} - \log_{10} T_{1/2,i}^{exp}. \tag{13}$$

A positive value of $E_{i,m}$ denotes an overprediction of the experimental half-life, whereas a negative value denotes an underprediction.

The absolute logarithmic error for each nucleus is

$$AE_{i,m} = |E_{i,m}|. \tag{14}$$

The overall root-mean-square deviation for model m is

$$RMS_m = \sqrt{\frac{1}{N}\sum_{i=1}^{N} E_{i,m}^2}, \tag{15}$$

where N is the number of nuclei in the common experimental validation dataset. RMS evaluation in logarithmic half-life space was also employed in the underlying study [24].

To provide information not contained in the aggregate RMS value alone, three complementary statistics were introduced in the present comparative re-analysis. The mean absolute logarithmic error is calculated as

$$MAE_m = \frac{1}{N}\sum_{i=1}^{N} |E_{i,m}|, \tag{16}$$

whereas the mean bias error is defined as

$$\mathrm{MBE}_m = \frac{1}{N}\sum_{i=1}^{N} E_{i,m}. \tag{17}$$

The MBE identifies directional model bias: $MBE > 0$ indicates an overall tendency to overpredict experimental half-lives, whereas $MBE < 0$ indicates an overall tendency to underpredict them.

Because RMS and MAE can be influenced by the magnitude of individual residuals, the median absolute logarithmic error is additionally evaluated as

$$\mathrm{MdAE}_m = \mathrm{median}\left(|E_{1,m}|, \dots, |E_{N,m}|\right). \tag{18}$$

The effect of the explicit neutron-skin treatment on each nucleus was quantified by

$$AE_{i,m} = AE_{i,conv} - AE_{i,skin}. \quad (19)$$

Accordingly, $\Delta AE_i > 0$ indicates that the neutron-skin-modified formulation provides the closer prediction, $\Delta AE_i < 0$ indicates that the conventional formulation provides the closer prediction, and $\Delta AE_i = 0$ denotes equivalent absolute logarithmic error. The number of nuclei satisfying each condition was subsequently determined to provide a simple model-win count independent of the aggregate RMS statistic. All error statistics reported in this study are evaluated in base-10 logarithmic half-life space; consequently, RMS, MAE, MBE, MdAE, $E_{i,m}$, $AE_{i,m}$, and AE_i are dimensionless quantities.

2.6. Previously Established and New Comparative Measures

A distinction was maintained between results reported in the two underlying studies and quantities calculated specifically for the present comparative re-analysis. The RMS deviations of 1.822 and 1.437 constitute previously reported results of the conventional DDCM–WKB [23] and neutron-skin-modified [24] formulations, respectively. In the present study, these independently reported RMS values were compared directly to quantify the relative change in overall predictive performance between the two formulations.

The percentage reduction in RMS deviation was calculated as

$$R_{RMS} = \frac{RMS_{conv} - RMS_{skin}}{RMS_{conv}} \times 100\%. \quad (20)$$

Using $RMS_{conv} = 1.822$ and $RMS_{skin} = 1.437$, the present comparative analysis gives

$$R_{RMS} = \frac{1.822 - 1.437}{1.822} \times 100\% \approx 21.1\%.$$

Thus, the individual RMS values are previously reported results from the respective source studies [23, 24], whereas the 21.1% relative reduction is a comparative result calculated in the present study.

In addition to the percentage RMS reduction, the nucleus-specific signed logarithmic residuals $E_{i,m}$, absolute logarithmic errors ($AE_{i,m}$), mean absolute logarithmic error (MAE), mean bias error (MBE), median absolute logarithmic error (MdAE), ΔAE_i, and model-win counts were evaluated in the present comparative re-analysis. These additional measures were introduced to determine whether the lower RMS deviation of the neutron-skin-modified formulation is supported by consistent improvements across individual nuclei or results primarily from improved predictions within a limited subset of the experimental data.

2.7. Criteria for Comparative Model Assessment

Model performance was assessed using complementary rather than isolated statistical measures. Lower RMS was interpreted as improved overall agreement with experiment, while lower MAE and MdAE indicated reduced average and typical nucleus-specific deviations, respectively. MBE was used to determine whether either formulation exhibited systematic overprediction or underprediction.

Nucleus-specific values of ΔAE_i were used to determine the direction and magnitude of the change in absolute logarithmic error produced by explicit neutron-skin treatment. The model-win count was used only as a descriptive indicator of consistency and was not interpreted as a statistical significance test.

Given the limited number of nuclei for which corresponding experimental spontaneous-fission half-lives are available in the source dataset, no claim of statistical significance or universal model superiority is made solely from the comparative error measures. Instead, the combined statistics are used to determine whether the reduction in RMS deviation obtained from comparison of the previously reported RMS values is supported by nucleus-level improvements and to identify exceptions requiring physical interpretation.

3. RESULTS AND DISCUSSION

3.1. Overall Comparative Performance

The comparative re-analysis indicates that explicit incorporation of neutron skin thickness improves the overall agreement of the calculated spontaneous-fission half-lives with the available experimental data, although the improvement is not uniform across all nuclei. The conventional DDCM–WKB and neutron-skin-modified formulations have previously reported root-mean-square (RMS) deviations of 1.822 [23] and 1.437 [24], respectively. Direct comparison of these independently reported values in the present study yields a reduction in RMS deviation of approximately 21.1%. The present analysis therefore examines how this aggregate difference in predictive performance is distributed across individual nuclei.

The common experimental subset contains nine nuclei for which corresponding experimental and calculated spontaneous-fission half-lives are available. In addition to the comparison of the previously reported RMS values, the present comparative re-analysis yields a mean absolute logarithmic error (MAE) of 1.587 for the conventional formulation and 1.255 for the neutron-skin-modified formulation. The corresponding median absolute logarithmic error (MdAE) decreases from 1.883 to 1.094. Thus, the neutron-skin-modified formulation exhibits reductions of approximately 20.9% in MAE and 41.9% in MdAE relative to the conventional formulation.

A substantial difference is also observed in model bias. The mean bias error (MBE) is −1.168 for the conventional formulation and −0.366 for the neutron-skin-modified formulation. The negative values indicate an overall tendency for both formulations to underestimate the experimental spontaneous-fission half-lives in base-10 logarithmic space. However, the substantially smaller magnitude of the MBE for the neutron-skin-modified formulation indicates that this systematic negative bias is markedly reduced.

Table 1. Comparative Performance of the Two Formulations Against Available Experimental Spontaneous-Fission Half-Lives.

Performance measure	Conventional DDCM–WKB	Neutron-skin-modified	Relative change
RMS	1.822	1.437	21.1% reduction
MAE	1.587	1.255	20.9% reduction
MBE	−1.168	−0.366	Reduced negative bias
MdAE	1.883	1.094	41.9% reduction
Nuclei more closely predicted	3/9 (33.3%)	6/9 (66.7%)	Modified model favoured overall

The agreement among RMS, MAE and MdAE provides stronger evidence of improved overall predictive performance than the RMS statistic alone. In particular, the substantial reduction in MdAE indicates that the improvement is not explained solely by a small number of extreme residuals. Nevertheless, aggregate statistics cannot establish universal superiority, making nucleus-specific comparison necessary.

3.2. Nucleus-Specific Performance and Model Bias

The experimental comparison includes ^{260}Lr, ^{262}Lr, ^{264}Lr, ^{262}Rf, ^{264}Rf, ^{266}Rf, ^{266}Db, ^{278}Bh, and ^{278}Hs. The underlying dataset provides experimental and corresponding theoretical spontaneous-fission half-lives for these nuclei.

The nucleus-specific absolute logarithmic errors show that the neutron-skin-modified formulation provides the closer prediction for six of the nine nuclei (66.7%): ^{262}Lr, ^{264}Lr, ^{264}Rf, ^{266}Rf, ^{266}Db, and ^{278}Hs. The conventional formulation remains closer to experiment for three nuclei (33.3%): ^{260}Lr, ^{262}Rf, and ^{278}Bh.

Table 2. Nucleus-Specific Absolute Logarithmic Errors.

Nucleus	AE_{conv}	AE_{skin}	ΔAE_i	**Closer formulation**
^{260}Lr	0.179	0.596	−0.417	Conventional
^{262}Lr	1.809	1.002	+0.808	Neutron-skin-modified
^{264}Lr	1.942	1.094	+0.847	Neutron-skin-modified
^{262}Rf	1.883	2.634	−0.750	Conventional
^{264}Rf	2.001	1.211	+0.790	Neutron-skin-modified
^{266}Rf	2.336	1.514	+0.822	Neutron-skin-modified
^{266}Db	2.920	2.154	+0.766	Neutron-skin-modified
^{278}Bh	0.083	0.769	−0.686	Conventional
^{278}Hs	1.125	0.319	+0.806	Neutron-skin-modified

Note: All error measures are evaluated in base-10 logarithmic half-life space and are therefore dimensionless.

Positive ΔAE denotes a reduction in absolute logarithmic error following explicit neutron-skin treatment, whereas negative ΔAE denotes deterioration relative to the conventional calculation. Among the improved cases, the largest reduction occurs for ^{264}Lr, where AE decreases from 1.942 to 1.094 ($\Delta AE_i = +0.847$). Comparable improvements occur for ^{266}Rf ($\Delta AE_i = +0.822$), ^{262}Lr ($+0.808$), ^{278}Hs ($\Delta AE_i = +0.806$), ^{264}Rf ($\Delta AE_i = +0.790$), and ^{266}Db ($\Delta AE_i = +0.766$). The relatively similar positive values of ΔAE_i indicate that the reduction in aggregate error is distributed across several nuclei rather than being dominated by one exceptionally improved prediction.

The exceptions are equally informative. For ^{260}Lr, the conventional calculation has an absolute logarithmic error of only 0.179, compared with 0.596 for the modified formulation. For ^{278}Bh, the conventional error is particularly small (0.083), whereas explicit neutron-skin treatment increases it to 0.769. The largest deterioration occurs for ^{262}Rf, where AE increases from 1.883 to 2.634.

The signed residuals explain part of this behaviour. The conventional formulation exhibits a pronounced overall tendency to underestimate the experimental half-lives, reflected in $\mathrm{MBE} = -1.168$. Explicit neutron-skin treatment reduces the magnitude of this bias to $\mathrm{MBE} = -0.366$. For nuclei substantially underestimated by the conventional formulation, the resulting change generally moves the calculated half-life closer to experiment. For example, the logarithmic residual changes from approximately −2.336 to −1.514 for ^{266}Rf, from −2.920 to −2.154 for ^{266}Db, and from −1.125 to −0.319 for ^{278}Hs.

However, the same shift can worsen predictions that are already close to experiment or positively biased. For ^{278}Bh, the residual changes from approximately −0.083 to +0.769, whereas for ^{262}Rf an existing positive residual of approximately +1.883 increases to +2.634. The explicit neutron-skin correction therefore improves the dominant underprediction tendency of the conventional model but can overcorrect particular nuclei.

This nucleus-dependent behaviour is important because it rules out the stronger claim that explicit neutron-skin treatment is universally superior. Instead, the results demonstrate improved overall and majority-case performance, accompanied by identifiable exceptions that require consideration of additional nuclear-structure effects.

3.3. Cluster-Radioactivity Behaviour

Cluster radioactivity exhibits considerably stronger nucleus-to-nucleus variation than the spontaneous-fission trends. In the underlying calculations, cluster-decay penetrabilities extend over many orders of magnitude, leading to similarly large variations in calculated half-lives and decay probabilities. This behaviour indicates that neutron skin thickness cannot independently account for the cluster-decay systematics.

In the conventional formulation, barrier penetrability generally decreases with increasing neutron skin thickness, while cluster-decay half-life correspondingly tends to increase. Considerable scatter is nevertheless observed across the investigated nuclei. The scatter is consistent with the simultaneous dependence of cluster emission on Q-value, emitted-cluster properties, reduced mass, Coulomb-barrier characteristics, shell effects, and daughter-nucleus configuration.

The physical role assigned to neutron skin differs between the two formulations. In the conventional approach [23], neutron skin thickness is principally examined in relation to the evolution of the calculated decay observables. In the modified formulation [24], neutron skin thickness enters explicitly into the modified matter radius and subsequently affects the surface separation and proximity interaction. The source formulation explicitly propagates the neutron-skin correction into the nuclear geometry and interaction potential.

Explicit neutron-skin treatment can therefore modify the geometrical conditions governing cluster tunnelling, but the calculated behaviour does not support neutron skin thickness as the sole control parameter for cluster radioactivity. Rather, neutron skin modifies one component of a multivariable tunnelling problem in which Q-value, barrier geometry, shell structure, and daughter-nucleus stability remain important.

An additional limitation must be emphasized. The present experimental validation dataset concerns spontaneous-fission half-lives; consequently, the comparative statements concerning cluster radioactivity represent a model-to-model theoretical comparison rather than an experimental demonstration that either formulation predicts cluster-decay half-lives more accurately. This distinction prevents the spontaneous-fission validation results from being generalized beyond the available experimental evidence.

3.4. Spontaneous-Fission Behaviour

Spontaneous fission displays a more systematic variation with neutron skin thickness within the modified formulation. Across the underlying dataset, neutron skin thickness increases from approximately $0.157\ fm$ for ^{260}Lr to $0.210\ fm$ for ^{322}Og.Over this range, the calculated spontaneous-fission barrier height decreases from approximately 5.296 to 4.303 MeV. The corresponding calculated half-life decreases from $7.0999 \times 10^{2}\ s$ to $8.6503 \times 10^{-6}\ s$, while the decay constant increases from $9.7627 \times 10^{-4}\ s^{-1}$ to $8.0130 \times 10^{4}\ s^{-1}$.

Within the WKB framework used in the model, these quantities are physically connected. A reduction in the calculated deformation barrier reduces the associated WKB action and consequently increases the calculated tunnelling probability. The underlying results similarly show increasing spontaneous-fission probability as the calculated barrier height and half-life decrease.

The comparative results are therefore consistent with the interpretation that explicit neutron-skin corrections alter the surface and geometrical terms entering the calculated fission barrier. However, the observed association between increasing neutron-skin thickness and decreasing fission barrier should not be interpreted as demonstrating that neutron-skin thickness independently causes the full evolution of spontaneous-fission lifetimes. Across the superheavy region, neutron-skin thickness varies concurrently with neutron excess, fissility, Coulomb repulsion, shell structure, deformation, and other nuclear properties.

The appropriate interpretation is therefore model-specific: explicit neutron-skin treatment provides an additional structural contribution to the calculated fission barrier and improves overall agreement with the available experimental half-lives, while other macroscopic and microscopic nuclear properties remain relevant to spontaneous-fission dynamics.

3.5. Competition Between Cluster Radioactivity and Spontaneous Fission

Both formulations indicate an evolution in the relative importance of cluster radioactivity and spontaneous fission across the investigated superheavy nuclei. In the conventional calculations, a crossover is observed from cluster-decay dominance at smaller neutron-skin thickness toward spontaneous-fission dominance at larger neutron-skin thickness. The neutron-skin-modified calculations retain the same general crossover behaviour.

The persistence of the crossover in both formulations is physically informative. It indicates that the transition is not generated solely by the explicit neutron-skin correction. Instead, the crossover accompanies the broader evolution of the investigated neutron-rich superheavy nuclei. Explicit neutron-skin treatment modifies the quantitative barrier characteristics and calculated lifetimes associated with that evolution.

The different behaviour of the two decay modes also contributes to the crossover. Cluster radioactivity remains highly sensitive to local nuclear-structure effects and consequently exhibits considerable scatter. Spontaneous fission shows a smoother decrease in calculated barrier height and half-life with increasing neutron-skin thickness in the modified formulation. The increasing competitiveness of spontaneous fission at larger neutron-skin thickness should therefore be understood as the combined outcome of changing surface structure, fissility, and other nuclear properties rather than as a single-variable neutron-skin effect.

This interpretation provides a useful distinction between neutron-skin thickness as an indicator and neutron-skin thickness as an explicit model parameter. The conventional approach establishes its association with the evolution of decay competition, whereas the modified formulation evaluates the consequences of allowing it to participate directly in the calculated nuclear geometry and barrier.

3.6. Implications and Limitations of Explicit Neutron-Skin Treatment

The principal contribution of the present comparative re-analysis is the demonstration that the RMS reduction obtained by directly comparing the previously reported RMS values of the two formulations is supported—but also qualified—by nucleus-resolved performance measures. The neutron-skin-modified formulation gives the closer prediction for six of the nine experimentally evaluated nuclei and simultaneously exhibits lower RMS, MAE, and MdAE, together with a reduced magnitude of the mean bias. The agreement among these measures provides stronger evidence of improved overall performance than the RMS comparison alone.

The reduction in MBE from −1.168 to −0.366 is particularly informative because it shows that an important component of the overall improvement arises from correction of the conventional formulation's systematic underprediction of the available spontaneous-fission half-lives. The nucleus-specific analysis, however, reveals that this correction is not universally beneficial. The poorer predictions for ^{260}Lr, ^{262}Rf, and ^{278}Bh demonstrate that explicit neutron-skin treatment alone cannot account for all nucleus-dependent variations in spontaneous-fission behaviour.

These exceptions point to limitations of the present formulation rather than invalidating its overall improvement. Spontaneous fission is intrinsically sensitive to shell corrections, deformation, pairing correlations, collective inertia, and other structural quantities in addition to surface and isospin effects. A neutron-skin-dependent correction may therefore improve the global representation of neutron-rich systems while remaining insufficient for nuclei whose decay dynamics are dominated by more localized structural effects.

The relatively small experimental validation subset also requires caution. The nine available spontaneous-fission cases are sufficient for a direct descriptive comparison of the two formulations but do not justify claims of universal predictive superiority across the entire superheavy region. Likewise, because equivalent experimental cluster-decay validation is not available in the present dataset, improved agreement for spontaneous fission should not be interpreted automatically as experimentally demonstrated improvement for cluster radioactivity.

Accordingly, the comparative evidence supports a specific rather than universal conclusion: within the investigated dataset, explicit incorporation of neutron skin thickness improves the overall prediction of available spontaneous-fission half-lives, principally by reducing systematic underprediction and improving six of the nine nucleus-specific comparisons, while retaining identifiable cases in which the conventional formulation performs better. This result provides a more rigorous assessment of the value and limitations of neutron skin thickness as an explicit structural parameter than can be obtained from the RMS reduction calculated from the previously reported RMS values alone.

4. CONCLUSION

This study presented a systematic comparative re-analysis of two previously developed approaches for describing cluster radioactivity and spontaneous fission in superheavy nuclei: the conventional DDCM–WKB framework and the unified neutron-skin-modified formulation. The objective was not to redevelop either model, but to determine quantitatively what is gained when neutron skin thickness is changed from a structural quantity associated with decay systematics to an explicit component of the nuclear geometry, interaction potential, and barrier-tunnelling formalism.

The comparison shows that explicit neutron-skin treatment improves the overall description of the available experimental spontaneous-fission half-lives. The conventional DDCM–WKB and neutron-skin-modified formulations have previously reported RMS deviations of 1.822 [23] and 1.437 [24], respectively. Direct comparison of these values in the present study corresponds to an RMS reduction of approximately 21.1%. The additional comparative analysis performed here supports this aggregate improvement: the mean absolute logarithmic error decreases from 1.587 to 1.255, the median absolute logarithmic error decreases from 1.883 to 1.094, and the mean bias error changes from −1.168 to −0.366. Taken together, these measures indicate that, within the investigated experimental validation subset, explicit neutron-skin treatment is associated with reduced overall prediction error and a substantially smaller systematic tendency to underestimate the available spontaneous-fission half-lives.

At the nucleus-specific level, the neutron-skin-modified formulation provides the closer prediction for six of the nine nuclei in the common experimental validation subset, whereas the conventional formulation performs better for ^{260}Lr, ^{262}Rf, and ^{278}Bh. The improvement is therefore supported by a majority of the individual comparisons but is not universal. These exceptions demonstrate that explicit neutron-skin treatment alone cannot account for all nucleus-dependent variations in spontaneous-fission behaviour and emphasize the continuing importance of shell structure, deformation, pairing correlations, collective effects, and other microscopic contributions.

The two decay modes also exhibit different sensitivities to neutron-rich surface structure. Cluster radioactivity displays pronounced nucleus-to-nucleus variation because its penetrability and half-life depend simultaneously on Q-value, barrier geometry, emitted-cluster properties, shell effects, and daughter-nucleus configuration. Neutron skin thickness should therefore be regarded as one structural contribution within a multivariable tunnelling problem rather than as an independent predictor of cluster emission. Spontaneous fission exhibits a more systematic evolution within the neutron-skin-modified formulation: across the investigated neutron-skin range, increasing neutron skin thickness is accompanied by decreasing calculated fission-barrier height and half-life and increasing decay constant. These trends establish the relevance of neutron-rich surface structure within the model while not implying that neutron skin thickness independently determines spontaneous-fission dynamics.

The persistence of the transition from cluster-radioactivity dominance toward spontaneous-fission dominance in both formulations further indicates that the crossover is not generated solely by the explicit neutron-skin correction. Instead, it accompanies the broader structural evolution of increasingly neutron-rich superheavy systems, while explicit treatment of neutron skin modifies the calculated geometry, barrier characteristics, and tunnelling probabilities associated with that evolution.

The neutron-skin-modified formulation should therefore be viewed as an extension of the conventional description rather than as a replacement for the other macroscopic and microscopic quantities governing the competing decay modes.

Overall, the present comparative re-analysis demonstrates that, within the investigated dataset, explicit incorporation of neutron skin thickness provides a more accurate global description of the available spontaneous-fission half-lives, principally by reducing systematic underprediction and improving six of the nine nucleus-specific comparisons. The remaining exceptions and the limited experimental validation sample preclude a claim of universal model superiority, while the absence of equivalent experimental cluster-decay validation means that the spontaneous-fission improvement should not be generalized automatically to cluster radioactivity. Extension to larger experimental datasets, together with systematic uncertainty and sensitivity analyses and more detailed treatment of shell, deformation, pairing, and collective effects, will be necessary to determine how broadly the observed improvement persists. The present results therefore identify neutron skin thickness as a physically meaningful structural input to superheavy-nuclear decay modelling while demonstrating that its predictive value is greatest when considered together with, rather than independently of, the other macroscopic and microscopic properties governing cluster radioactivity and spontaneous fission.

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Conflict of interest

The authors declare that they have no relevant financial or non-financial competing interests.

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Data availability

The data analyzed in this study was obtained from the previously published studies cited as Refs. [23] and [24], together with the nuclear and experimental data sources described in the Methods section. The comparative quantities generated in the present study are reported within the paper and its tables.

Ethics Approval

Not applicable.

Consent to Participate

Not applicable.

Consent for Publication

Not applicable.

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