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Integration of Monte Carlo and Hybrid Trisection-False Position Methods for Global Root-Finding of Nonlinear Equations

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ABSTRACT

Root-finding for nonlinear equations is a fundamental problem in numerical analysis. Classical bracketing methods, such as the bisection and false position methods, exhibit strong stability but rely heavily on the selection of an initial interval that contains a root. This study proposes an integration of the Monte Carlo method and a hybrid trisection–false position method for the global solution of nonlinear equations. The Monte Carlo method is employed to identify subintervals containing roots, including multiple roots, while the hybrid trisection–false position method is utilized to accelerate convergence toward the desired solution. Numerical experiments were conducted using Python programming on several nonlinear functions with 3,000 random samples. The results demonstrate that the proposed approach can effectively detect root-containing intervals and obtain accurate root approximations with a relatively small number of iterations. Therefore, the proposed method provides an effective alternative for the global solution of nonlinear equations.

***Keywords*:** Nonlinear equations, root-finding, Monte Carlo method, hybrid trisection-false position method, global root search.

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1. INTRODUCTION

Before the advent of digital computers, root-finding problems were solved using various approaches. For simple equations, roots could be obtained analytically. However, most equations arising in science, mathematics, and real-world engineering problems cannot be solved analytically due to their nonlinear and complex nature, thus requiring numerical approaches [1], [2], [3]. Methods that emerged at that time, such as graphical methods and trial-and-error methods, could only provide rough approximations, were inefficient, and were inadequate for engineering applications [2].

One of the methods developed for root-finding is the bracketing method. This method begins by estimating an interval (bracket) containing a root and then systematically reducing the length of that interval. Two specific methods that evolved from this approach are the bisection method and the false position method. This method guarantees the existence of the root [4], but exhibits a slow convergence rate [5]. In addition to bracketing methods, methods that do not require an initial interval enclosing a root were also developed, commonly referred to as open methods. Some well-known examples are the Newton–Raphson method and the secant method. These methods are computationally more efficient but do not always guarantee successful root detection [2].

Numerous studies have focused on optimizing the local convergence rate through the combination of numerical methods or algorithms. The BF (Bisection–False Position) and FB (False Position–Bisection) modifications proposed by Sabharwal demonstrated that such combinations could achieve a superlinear convergence rate superior to that of the pure bisection method [6]. In subsequent work, Sabharwal further combined the Bisection–False Position approach with the Newton–Raphson method [7]. Similarly, Vivas-Cortez et al. modified the bisection and modified false position approaches [8]. Suhadolnik developed combined bracketing methods that integrate bisection, false position, and parabolic interpolation to improve convergence speed while preserving global convergence guarantees [9]. Sharma and Chouhan proposed a hybrid Newton–Secant method that exhibited better computational performance than classical methods across various real-world nonlinear models [10]. Furthermore, Badr et al. developed a hybrid algorithm that combines the advantages of the trisection and false position methods [11]. Numerically, this algorithm outperformed the secant method, the pure trisection method, the Newton–Raphson method, and other bisection-based hybrid methods in terms of both the number of iterations and average execution time. Several other studies have also developed hybrid methods that combine the strengths of open methods and bracketing methods. Hitti et al. introduced always-convergent hybrid methods that integrate Newton's and Chebyshev's methods with the bisection method, resulting in faster algorithms while maintaining convergence guarantees [12]. A similar approach was proposed by Comemuang and Janngam through the integration of the Newton–Raphson and bisection methods to improve the stability of the iterative process for nonlinear equations [13]. Despite these advances, these hybrid methods still suffer from limitations because they remain local in nature and strongly depend on the selection of an appropriate initial interval to ensure successful root-finding.

A significant recent innovation was introduced through the Improved Bisection Method based on Slope (IBMS) by Etesami et al., which integrates a Monte Carlo sampling technique to randomly map the entire function domain. This stochastic approach enables the global identification of root locations, including double roots (roots that merely touch the x-axis), which are often undetected by the sign-change criterion employed in the classical bisection method [14].

Recent studies on nonlinear root-finding methods have also revealed a research trend toward combining global search capability with accelerated local convergence [15]. This trend arises because most modern hybrid methods still rely on the availability of a suitable initial interval to guarantee successful iterations.

To overcome the limitations of conventional bracketing methods, this study proposes an integrative algorithm that operates in two systematic stages. The first stage employs the Monte Carlo method to stochastically scan the entire function domain and identify root locations without requiring a manually specified search interval. The second stage integrates the hybrid trisection–false position method to accelerate superlinear convergence within the identified subintervals. The proposed approach is expected to combine the flexibility of global root search with high computational efficiency.

2. MATERIALS AND METHODS

2.1. Trisection Method

The trisection method divides the interval $[a, b]$ into three equal subintervals and searches for a root in the subinterval whose endpoint function values have opposite signs. To divide the interval $[a, b]$ into three equal parts using x_1 and x_2, consider Figure 1.

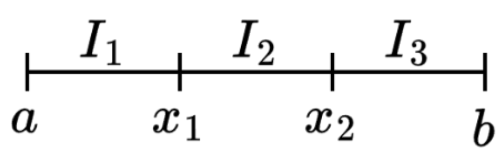

Figure 1. Division of An Interval Into Three Subintervals.

with $I_1 = [a, x_1]$, $I_2 = [x_1, x_2]$, and $I_3 = [x_2, b]$.
We obtain

$$x_1 - a = b - x_2 \tag{1}$$

and

$$x_2 - x_1 = x_1 - a \tag{2}$$

From equations (1) and (2), it follows that

$$x_1 = \frac{2a + b}{3},$$

and

$$x_2 = \frac{2b + a}{3}.$$

The length of the interval $[a, b]$ is reduced to one-third of its previous size at each iteration [11]. Badr et al. showed that the trisection method requires fewer theoretical iterations,

$$n = \left\lceil \log_3 \left(\frac{b-a}{\varepsilon} \right) \right\rceil$$

than the bisection method,

$$n = \left\lceil \log_2 \left(\frac{b-a}{\varepsilon} \right) \right\rceil$$

indicating that the trisection method can reduce the search interval more efficiently than the bisection method.

2.2. False Position Method

The false position method is a dynamic and efficient method when the function behavior is nonlinear. The function $f(x)$, whose root is to be determined within the interval $[a, b]$, must be continous, and the function values at the endpoints of the interval $[a, b]$ must have opposite signs. The false position method uses two endpoint values of the interval, $(r_0 = a, r_1 = b)$. The line connecting the two points $\left(r_0, f(r_0)\right)$ and $\left(r_1, f(r_1)\right)$ intersects the x-axis at the next approximation (r_2). For determining subsequent approximations (r_n), the following formula is used:

$$r_n = r_{n-1} - \frac{f(r_{n-1})(r_{n-1} - r_{n-2})}{f(r_{n-1}) - f(r_{n-2})}$$

for $n \geq 2$ [11].

2.3. Blended Trisection and False Position

Badr et al. [11] exploited the advantages of the trisection method over the bisection method to develop a new hybrid approach that combines the trisection and false position. The trisection method is employed to reduce the root-search interval by dividing the interval $[a, b]$ into three equal parts. Meanwhile, the false position method is used to obtain a root approximation through linear interpolation. The combination of these two methods provides better performance than several classical methods, including the bisection, false position, Newton–Raphson, and secant methods.

The hybrid trisection–false position method utilizes information obtained from both the trisection and false position methods to update the search interval at each iteration. The iterative process continues until the prescribed error tolerance is satisfied.

2.4. Monte Carlo Method

In the context of nonlinear equation root-finding, Etesami et al. [14] employed the Monte Carlo method to divide the desired search domain into smaller regions. In this approach, a number of random points are generated over the function domain using a uniform distribution. These points are then used to evaluate sign changes in the function values as well as variations in the function slope over the search domain. This approach enables the detection of tangent roots, namely roots that merely touch the x-axis without changing the sign of the function, which are difficult to detect using the conventional bisection method because it relies on sign changes within the initial interval.

Furthermore, the use of Monte Carlo sampling enables root-finding to be performed globally over the entire function domain. As the number of random samples increases, the probability of identifying root-containing subintervals also increases. This approach is highly advantageous because it eliminates dependence on initial guesses, thereby allowing all roots of a nonlinear equation to be detected simultaneously within a single execution.

3. RESULT AND EXPERIMENTAL

3.1. Proposed Method

This study was conducted to develop a nonlinear equation root-finding method by integrating the Monte Carlo method with the hybrid trisection–false position method. The procedure begins by specifying the nonlinear function, the search interval, the number of random samples, and the error tolerance. Subsequently, the Monte Carlo method is employed to perform stochastic sampling over the function domain in order to identify subintervals that contain roots without requiring manually selected initial intervals. The identified subintervals are then used as initial intervals for the hybrid trisection–false position method to obtain root approximations with faster convergence.

During the hybrid trisection–false position stage, the root interval is iteratively refined using both the trisection and false position methods until the prescribed error tolerance is satisfied. The final output consists of root approximations, which are subsequently analyzed in terms of the number of iterations required and the effectiveness of the method in detecting roots, including tangent roots. The overall procedure of the proposed method is illustrated in Figure 2.

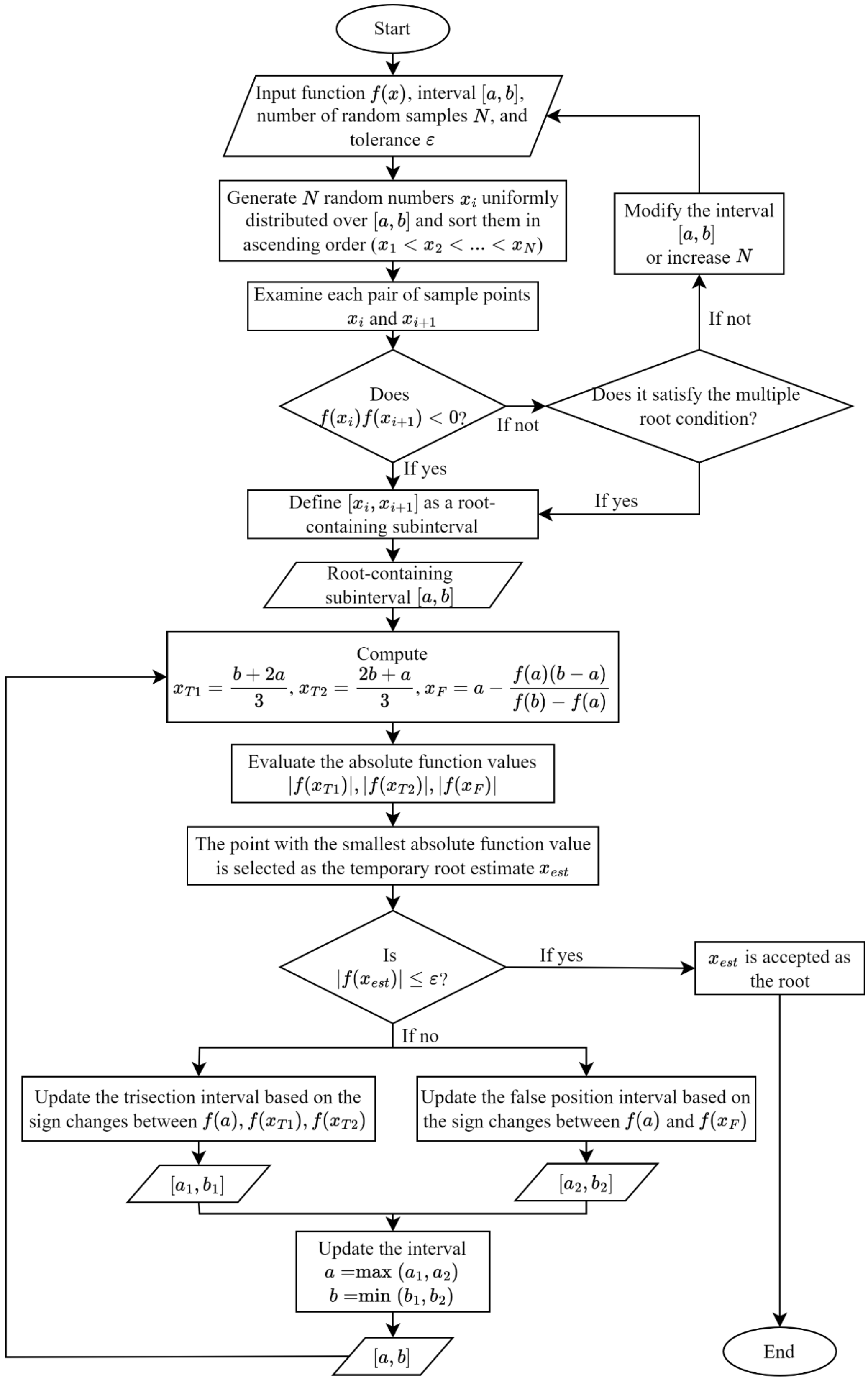

Figure 2. Flowchart of the Proposed Monte Carlo-Trisection-False Position Hybrid Algorithm.

3.2. Experimental Results

The experiments were conducted using the functions and intervals listed in Table 1.

Table 1. Test Functions and Intervals.

No.	Function	Interval
1	$x\cos^2(x)$	$[-2,2]$
2	x^2-3	$[1,2]$
3	x^2-5	$[2,7]$
4	e^x-3x-2	$[2,3]$
5	$\sin(x)-x^2$	$[-1,1]$
6	xe^x-1	$[-3,3]$
7	$(x-2)^2-\ln(x)$	$[1,5]$
8	$e^x-1-\cos(x)$	$[-3,5]$
9	$(x-4)(x-1)^2$	$[0,5]$
10	$(x+1)(x-5)(x-2)^2$	$[-2,6]$

Numerical computations were performed using Python programming with 3000 randomly generated sample points and a tolerance value of $\varepsilon = e^{-10}$. The detailed results are presented in Table 2.

Table 2. Numerical Results of the Proposed Monte Carlo-Trisection-False Position Method.

Function	Approximate Root	Iterations	Absolute Error
$x\cos^2(x)$	$-1{,}571$	3	$3{,}93e^{-11}$
	0	1	$7{,}73e^{-11}$
	$1{,}571$	3	$3{,}93e^{-11}$
x^2-3	$1{,}732$	2	$5{,}83e^{-12}$
x^2-5	$2{,}236$	2	$2{,}35e^{-12}$
e^x-3x-2	$2{,}125$	3	$4{,}8e^{-14}$
$\sin(x)-x^2$	0	2	$4{,}04e^{-11}$
	$0{,}877$	2	$1{,}57e^{-11}$
xe^x-1	$0{,}567$	3	$1{,}55e^{-12}$
$(x-2)^2-\ln(x)$	$1{,}412$	3	$5{,}13e^{-14}$
	$3{,}057$	2	$1{,}1e^{-11}$
$e^x-1-\cos(x)$	$-2{,}789$	3	$5{,}21e^{-13}$
	$0{,}601$	3	$1{,}04e^{-13}$
$(x-4)(x-1)^2$	$0{,}999$	2	$7{,}5e^{-11}$
	4	2	$8{,}27e^{-11}$
$(x+1)(x-5)(x-2)^2$	-1	3	$2{,}31e^{-12}$
	$1{,}999$	3	$2{,}25e^{-10}$
	5	2	$3{,}46e^{-11}$

From the numerical results, it can be observed that the proposed integrated method successfully identifies intervals containing roots, including multiple roots. Furthermore, the root-finding process can be considered computationally efficient, as only a small number of iterations are required to achieve convergence.

4. CONCLUSIONS

This study successfully developed an integrative algorithm that combines the Monte Carlo method and the hybrid trisection–false position method for solving nonlinear equations. The Monte Carlo method was able to automatically detect subintervals containing roots, including tangent roots that are often difficult to identify using conventional bracketing methods. Subsequently, the hybrid trisection–false position method exhibited rapid convergence, requiring only a relatively small number of iterations. The experimental results demonstrate that the integration of these two methods effectively combines global search capability with local convergence efficiency, making it a promising alternative for solving nonlinear equations.

As a direction for future research, this integrative methods approach can be extended to solve multi-variable system of nonlinear equations with a higher degree of complexity. In addition, the performance of the algorithm during the global search phase can be further optimized by refining its stochastic sampling techniques to reduce the computational time overhead. Furthermore, subsequent studies could explore various other global root-finding methods to provide a mor comprehensive and objective performance comparison against the proposed algorithm.

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