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Comparative Study of Fourth-Order Runge-Kutta and Adaptive Runge-Kutta-Fehlberg (RKF45) Methods for Solving Logistic Population Models

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ABSTRACT

This study evaluates and compares the performance of the fourth-order Runge-Kutta (RK4) method against the adaptive Runge-Kutta-Fehlberg 45 (RKF45) method in solving a logistic growth model to predict population dynamics in West Java Province. Growth rate parameters and carrying capacities were estimated through curve-fitting optimization based on historical demographic data from Statistics Indonesia. The performance of both numerical schemes was rigorously assessed using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) metrics. The empirical results demonstrate that the adaptive RKF45 method significantly outperforms the fixed-step RK4 approach in terms of precision, yielding substantially lower error rates. While the overall predictive accuracy remains robust, the logistic model exhibited minor deviations in response to abrupt population fluctuations caused by the COVID-19 pandemic. Ultimately, the inherent flexibility of RKF45 in dynamically adjusting step sizes renders it a highly recommended instrument for accurate demographic forecasting. Implementing such computational methods provides a strategic framework for regional governments in formulating infrastructure planning and promoting sustainable development for the future.

Keywords: RK4, RKF45, logistic model, population growth.

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1. INTRODUCTION

Indonesia is ranked as one of the most populous countries globally, securing the fourth position behind China, India, and the United States [1]. This massive population size elevates demographic issues into a critical aspect of national development, given that population growth dynamics are intricately linked to various social and economic challenges that policymakers must anticipate [1]. Furthermore, Indonesia is undergoing demographic shifts characterized by altering fertility patterns, an increasing age at first marriage, and a diversifying family structure, as conceptualized within the framework of the second demographic transition [2]. At the regional level, West Java Province has exhibited a consistent and continuous increase in population size, which poses potential challenges across social, health, and environmental dimensions [3]. Concurrently, the rapid pace of urbanization on Java Island has driven the expansion of built-up areas and the formation of sprawling urban agglomerations, thereby introducing complex challenges in cross-administrative boundary regional planning [4]. This dynamic is also closely tied to the pivotal role of urban areas as epicenters of economic growth, despite persisting structural constraints that may hinder productivity [5]. Consequently, a comprehensive understanding of population dynamics and projections is paramount to serve as a foundational basis for formulating effective and sustainable development policies.

Within this context, the logistic growth model, first introduced by Pierre François Verhulst in 1838, represents a widely utilized mathematical approach for modeling population dynamics under resource-constrained conditions [3]. This model operates on the assumption that population growth is initially rapid but subsequently decelerates as population density increases and environmental resources become limited, ultimately asymptotically approaching the carrying capacity [6, 7]. Mathematically, these dynamics are formulated via the nonlinear differential equation $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$, which integrates the intrinsic growth rate with environmental limiting factors, thereby offering a more realistic representation of population behavior compared to exponential models. The logistic model has been extensively applied in demographic data analysis and has proven capable of providing a robust fit across various empirical contexts [8]. In the context of West Java, this model is implemented using census data to estimate population growth and future projections, with model selection predicated on minimizing prediction error metrics, such as the Mean Absolute Percentage Error (MAPE) [3]. Furthermore, to accommodate data uncertainties, a fuzzy-based approach is employed and solved numerically using the fourth-order Runge-Kutta method, demonstrating that the influence of uncertainty tends to diminish over the long term, thereby confirming the structural robustness of the model [3].

Selecting an appropriate numerical method constitutes a crucial factor in enhancing the accuracy of population projections. Due to its nonlinear nature, the logistic model frequently defies analytical solution, thereby necessitating numerical approaches such as the Runge-Kutta (RK) methods. The fourth-order Runge-Kutta (RK4) method is widely recognized for its optimal balance between computational accuracy and ease of implementation [9]. This method has been successfully deployed across a diverse spectrum of applications, ranging from human population growth modeling to interspecies interactions in plant biology [10, 11]. Nevertheless, the utilization of a fixed step size in RK4 presents inherent limitations, particularly when applied to highly dynamic systems or extended long-term projections. Under such conditions, it is susceptible to accumulating truncation errors and may exhibit numerical instability if the selected step size is sub-optimal [9, 12].

As an alternative, the Runge-Kutta-Fehlberg (RKF45) method offers an adaptive approach by automatically adjusting step sizes based on local error estimates. The efficacy of this method has been demonstrated in several studies, such as population growth predictions in West Sulawesi [13] and population projections in Nigeria [14], both of which yielded remarkably low relative errors. A comparative study by Satar and Arifin [12] on cell growth models also indicated that the adaptive RKF45 exhibits higher accuracy than the fixed-step RK4. Nevertheless, a distinct research gap persists, as no prior study has systematically and directly compared the performance of RK4 and RKF45 within a logistic model framework for human population projections. A study by Azis et al. [15] only compared RK4 with the Adams-Bashforth-Moulton method, rather than with RKF45. Therefore, this study aims to bridge this gap by conducting a comparative analysis of both methods in solving the logistic growth model, thereby determining which scheme is more accurate, stable, and efficient for future population projections.

2. MATERIALS AND METHODS

This section describes the population data of West Java Province used as the basis for modeling, along with an explanation of the population growth rate and carrying capacity, logistic equation, and two numerical methods applied, namely Fourth Order Runge-Kutta method (RK4) and Runge-Kutta Fehlberg 45 method (RKF45).

2.1. Population Data

A population is the total number of individuals inhabiting a given area during a specific period, at the local, national, or global scale. A population encompasses diversity in age groups, ethnic backgrounds, religions, and social status, as well as diverse social and economic dynamics. Population is a key aspect of demographic studies because it influences government policies, the economy, and a country's infrastructure development. Therefore, population growth projections are essential for gaining a deep understanding of demographic changes, including in planning for sustainable development and the equitable distribution of resources.

The population data used in this study consists of population figures for West Java Province, Indonesia, from 2010 to 2025, expressed in thousands. The data is sourced from Statistics Indonesia and is presented in Table 1 below.

Table 1. Population of West Java Province.

Year	Population (thousand people)
2010	43,227.107
2011	43,938.796
2012	44,643.586
2013	45,340.799
2014	46,029.668
2015	46,709.569

2016	47,379.389
2017	48,037.827
2018	48,683.7
2019	49,316.7
2020	48,152.27
2021	48,738.83
2022	49,306.8
2023	49,860.33
2024	50,345.19
2025	50,759

It should be noted that the data for 2020–2021 show a downward population trend that is likely due to the COVID-19 pandemic. The logistic model used in this study assumes continuous growth without external shocks, so the predictions for that period may contain greater deviations.

2.2. Population Growth Rate and Population Carrying Capacity in West Java

The population growth rate (r) is a measure that describes how quickly or slowly a population in a given area increases or decreases over a specific period of time. Carrying capacity (K) is the maximum number of individuals that an environment can support over a specific period of time without experiencing environmental degradation or resource depletion. The values of r and K are determined using a curve-fitting method that minimizes the sum of squared errors between the logistic model and the actual data:

$$\min_{r,K} \sum_{i=1}^{n} \left(P_{actual,i} - P_{model,i}(r,K)\right)^2 \tag{1}$$

Based on West Java Province population data for 2010–2025, the optimization results obtained are:

$$r = 0.11081103 \text{ per year and } K = 52{,}431.774 \text{ thousand people} \tag{2}$$

with an initial population value of:

$$P_0 = 43{,}227.107 \text{ thousand people} \tag{3}$$

2.3. Logistic Population Growth Model

Logistic equation is a mathematical model used to describe population growth while taking into account the limits of the environment's carrying capacity. This model was first introduced by Pierre-François Verhulst in 1838. Logistic equation is written as below:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{4}$$

with the initial condition:

$$P(0) = P_0 = 43{,}227.107 \text{ thousand people} \tag{5}$$

Based on the values of r and K obtained previously, the logistic equation for this study becomes:

$$\frac{dP}{dt} = 0.11081103P\left(1 - \frac{P}{52{,}431.774}\right) \tag{6}$$

2.4. Fourth Order Runge-Kutta Method (RK4)

The RK4 method is one of the most popular numerical methods for solving ordinary differential equations (ODEs). This method offers a good balance between computational accuracy and efficiency, making it a common choice in numerical simulations. The advantage of this method lies in its ability to achieve fourth-order accuracy without the need to involve higher-order derivatives. This method is a valuable tool in modeling and simulating various dynamic systems involving differential equations. For the differential equation $\frac{dP}{dt} = f(t, P)$ with step size h, RK4 method is given by [12]:

$$k_1 = hf(t_n, P_n) \tag{7}$$

$$k_2 = hf\left(t_n + \frac{h}{2}, P_n + \frac{k_1}{2}\right) \tag{8}$$

$$k_3 = hf\left(t_n + \frac{h}{2}, P_n + \frac{k_2}{n}\right) \tag{9}$$

$$k_4 = hf(t_n + h, P_n + k_3) \tag{10}$$

$$P_{n+1} = P_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{11}$$

In this study, the RK4 method was tested with four variations of step sizes to observe convergence behavior and sensitivity to step size selection, namely:

- $h = 1$ (1 year)
- $h = 0.5$ (6 months)
- $h = 0.1$ (approximately 36 days)
- $h = 0.01$ (approximately 3.65 days)

2.5. Runge-Kutta Fehlberg 45 Method (RKF45)

The RKF45 method is an adaptive numerical method that combines 4th-order and 5th-order Runge-Kutta methods to estimate the local error and adjust the step size automatically. This method was developed by Erwin Fehlberg in 1969. With its adaptive step size capability, RKF45 is able to maintain high accuracy with better computational efficiency compared to fixed step size methods. The formulation of the RKF45 method requires six coefficient values for each step, namely [12]:

$$k_1 = hf(t_n, P_n) \tag{12}$$

$$k_2 = hf\left(t_n + \frac{1}{4}h, P_n + \frac{1}{4}k_1\right) \tag{13}$$

$$k_3 = hf\left(t_n + \frac{3}{8}h, P_n + \frac{3}{32}k_1 + \frac{9}{32}k_2\right) \tag{14}$$

$$k_4 = hf(t_n + \frac{12}{13}h, P_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3 \tag{15}$$

$$k_5 = hf\left(t_n + h, P_n + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right) \tag{16}$$

$$k_6 = hf\left(t_n + \frac{1}{2}h, P_n - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right) \tag{17}$$

The 4th-order solution of the RKF45 method is expressed as:

$$P_{n+1} = P_n + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5 \tag{18}$$

Meanwhile, the 5th-order solution is expressed as:

$$z_{n+1} = P_n + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6 \tag{19}$$

Local error estimate is calculated as the difference between the 5th-order solution and the 4th-order solution, namely:

$$Error = |z_{n+1} - P_{n+1}| \tag{20}$$

New step size is determined based on the following scaling factor:

$$h_{new} = 0.84 \cdot h \cdot \left(\frac{tol \cdot h}{Error}\right)^{\frac{1}{4}} \tag{21}$$

The step acceptance criteria in the RKF45 method are as follows:

- If $Error \leq tol$: the step is accepted, proceed to the next iteration.
- If $Error > tol$: the step is rejected, repeat with a smaller step size.

Initial step size (h_0) is set to 0.1 years (approximately 36 days), with varying tolerances according to the experimental scenarios. The iteration is stopped when the simulation time reaches $t = 15$ years (period 2010–2025).

In this study, the RKF45 method was tested with four variations of tolerance values to observe the effect of tolerance on accuracy and computational efficiency, namely:

- $tol = 1 \times 10^{-3}$
- $tol = 1 \times 10^{-4}$
- $tol = 1 \times 10^{-6}$
- $tol = 1 \times 10^{-8}$

2.6. Model Evaluation

The evaluation of the numerical method's performance in this study is carried out by calculating the error between the predicted results and the actual data. The error metrics used are the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE).

MAE measures the average absolute difference between predicted and actual values. MAE is defined as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|P_{actual,i} - P_{predicted,i}\right| \tag{22}$$

where:

- n: number of data points
- $P_{actual,i}$: actual population in the i-th year
- $P_{predicted,i}$: predicted population in the i-th year

MAE provides an overview of the average error in the same unit as the original data (thousands of people). The smaller the MAE value, the more accurate the prediction model.

RMSE measures the square root of the average squared difference between predicted and actual values. RMSE is defined as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(P_{actual,i} - P_{predicted,i}\right)^2} \tag{23}$$

RMSE is more sensitive to large errors than MAE because the squaring process amplifies the contribution of significant errors. This makes RMSE useful for detecting anomalies or large deviations in predictions.

3. RESULTS AND DISCUSSION

3.1. Numerical Simulation Results and Comparison of RK4 and RKF45 Methods

RK4 method was tested with four step size variations ($h = 1, 0{,}5, 0.1, 0.01$) using parameters $r = 0.11081103$ per year and $K = 52{,}431.774$ thousand people. The simulation results are presented in Table 2.

Table 2. Performance of the RK4 Method with Step Size Variations.

Step Size (h)	MAE (thousand)	RMSE (thousand)	Total Iterations	Runtime (ms)
1.0	277.3376	371.2286	15	0.1348
0.5	277.3377	371.2286	30	0.1590
0.1	277.3377	371.2286	150	0.4953
0.01	277.3377	371.2286	1500	4.3266

Based on Table 2, the RK4 method shows very consistent performance across all step size variations. The resulting RMSE and MAE values are relatively similar, at approximately 371.23 thousand people and 277.34 thousand people, indicating that this method is stable against changes in step size.

However, reducing the step size significantly increases the number of iterations from 15 to 1500 iterations and the computation time from 0.13 ms to 4.33 ms, without any improvement in accuracy. Therefore, using a step size of $h = 1$ year is the most efficient choice for the RK4 method in predicting population growth.

RKF45 method was tested with four tolerance variations $tol = 10^{-3}, 10^{-4}, 10^{-6}, 10^{-8}$. The simulation results are presented in Table 3.

Table 3. Performance of the RKF45 Method with Tolerance Variations.

Tolerance (tol)	MAE (thousand)	RMSE (thousand)	Total Iterations	Runtime (ms)
10^{-3}	273.9098	369.4707	11	0.1281
10^{-4}	276.5478	371.0586	16	0.1635
10^{-6}	277.2156	371.1615	48	0.4410
10^{-8}	277.3239	371.2239	147	1.3258

Based on Table 3, the RKF45 method shows that the tighter the tolerance value, the accuracy does not always increase monotonically. A tolerance of 10^{-3} actually produces the smallest error with RMSE = 369.4707 thousand people, MAE = 273.9098 thousand people, while tighter tolerances such as $10^{-4}, 10^{-6}$ and 10^{-8} actually produce slightly larger errors. This indicates that choosing an excessively tight tolerance does not provide accuracy benefits, but instead significantly increases the number of iterations and computation time. From an efficiency perspective, a tolerance of 10^{-3} completes the simulation in only 11 iterations with a time of 0.1281 ms, making it the most optimal choice for balancing accuracy and computational speed.

Comparison of the best methods from RK4 and RKF45 is presented in Table 4.

Table 4. Performance of the RKF45 Method with Tolerance Variations.

Method	Parameter	MAE (thousand)	RMSE (thousand)	Total Iterations	Runtime (ms)
RK4	$h = 1.0$	277.3376	371.2286	15	0.1348
RKF45	$tol = 10^{-3}$	273.9098	369.4707	11	0.1281

Based on Table 4, the RKF45 method with a tolerance of 10^{-3} is proven to be superior to the RK4 method with a step size of $h = 1.0$ in predicting population growth in West Java Province. The RKF45 method produces RMSE of 369.4707 thousand people and MAE of 273.9098 thousand people, which are lower compared to RK4, which recorded RMSE of 371.2286 thousand people and MAE of 277.3376 thousand people. In addition to being more accurate, RKF45 is also more efficient as it requires only 11 iterations and a computation time of 0.1281 ms. Thus, the RKF45 method is not only superior in accuracy but also in computational efficiency, making it the best choice for long-term population projections.

3.2. Visualization of the Comparison between RK4 and RKF45

Based on the evaluation results that have been conducted, the RKF45 method with a tolerance of 10^{-3} was selected as the best method because it produced the smallest error compared to the RK4 method. Figure 1 presents a visual comparison between the actual data, the predictions of the best RK4 method ($h = 1.0$), and the predictions of the best RKF45 method ($tol = 10^{-4}$)

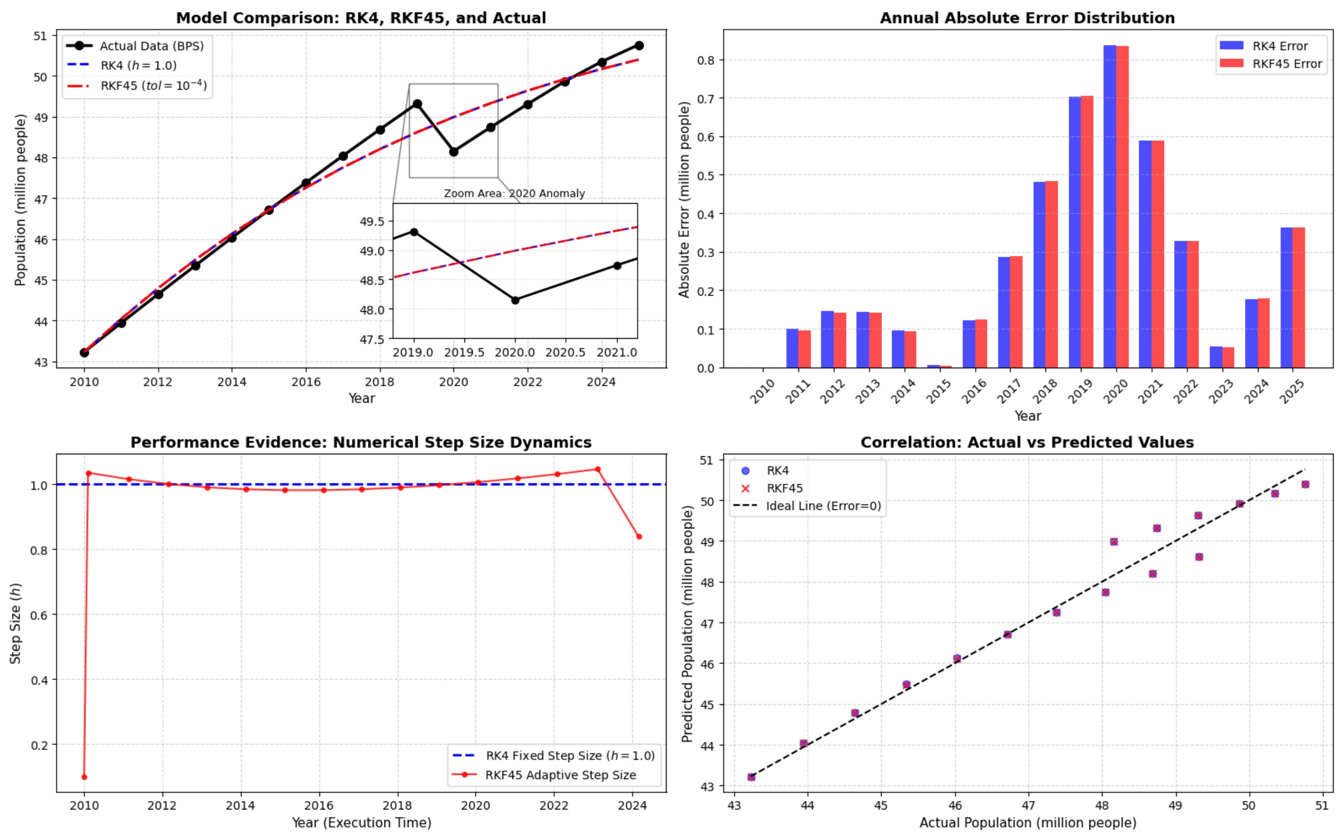

Figure 1. Comparison of Actual Data with Predictions from RK4 and RKF45 Methods.

Figure 1 displays the population growth curves predicted by RK4 and RKF45 compared to the actual Statistics Indonesia data for the 2010-2025 period. In general, both methods are able to follow the growth pattern. However, in the circled area (Zoom Area: 2020 Anomaly), it can be seen that the RKF45 method with adaptive steps is better able to approach the actual data compared to RK4, which uses a fixed step size. This demonstrates the superiority of RKF45 in responding to unexpected data fluctuations, such as the population decline due to the COVID-19 pandemic.

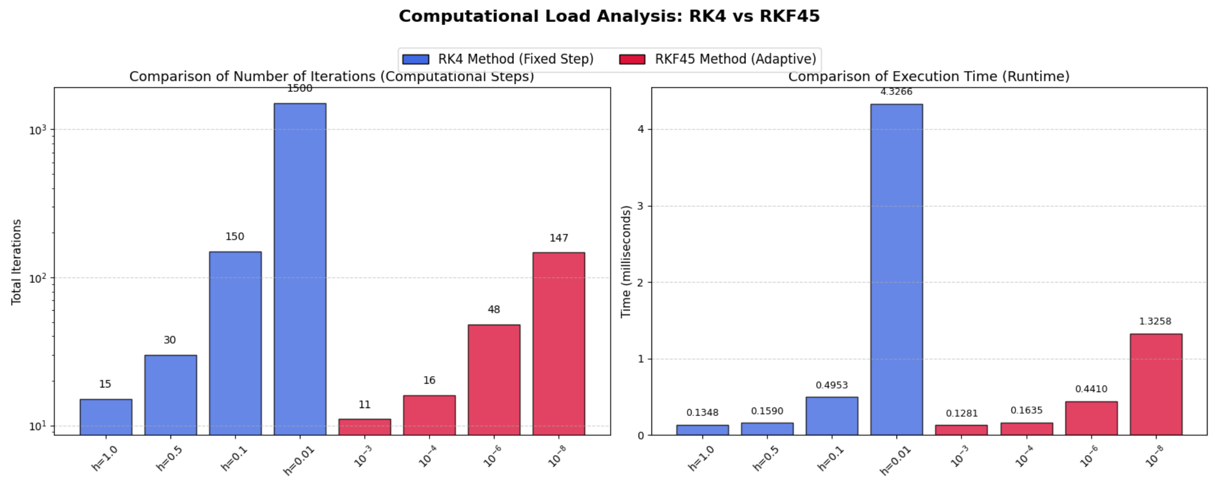

Figure 2. Comparison of Number of Iterations and Execution Time Between Methods.

Figure 2 presents a comparison of the number of iterations and execution time between methods. RK4 method with a fixed step size shows that the smaller the h, the number of iterations increases drastically, from 15 iterations at $h = 1.0$ to 1500 iterations at $h = 0.01$, with execution time increasing up to 4.33 ms. In contrast, RKF45 method with an adaptive approach is able to complete the simulation in only 11–16 iterations, with an execution time of 0.12–0.17 ms for tolerances of 10^{-3} to 10^{-4}. Even at the tightest tolerance of 10^{-8}, the iterations of RKF45 with 147 iterations and 1.33 ms are still far more efficient compared to RK4 with $h = 0.01$.

3.3. Annual Prediction Results of the Best Method

RKF45 method with a tolerance of 10^{-3} was selected as the best method. The annual prediction results are presented in Table 5.

Table 3. Population Prediction Results using the RKF45 Method $tol = 10^{-3}$.

Year	Actual (thousand people)	Prediction (thousand people)	Absolute Error	Relative Error
2010	43227.107	43227.107	0.0000	0.0000%
2011	43938.796	44014.638	75.8416	0.1726%
2012	44643.586	44784.777	141.1910	0.3163%
2013	45340.799	45465.070	124.2711	0.2741%
2014	46029.668	46114.643	84.9747	0.1846%
2015	46709.569	46702.069	7.4998	0.0161%
2016	47379.389	47240.955	138.4335	0.2922%
2017	48037.827	47745.570	292.2568	0.6084%
2018	48683.700	48187.025	496.6746	1.0202%

2019	49316.700	48609.603	707.0969	1.4338%
2020	48152.270	48975.497	823.2271	1.7096%
2021	48738.830	49319.217	580.3868	1.1908%
2022	49306.800	49627.772	320.9720	0.6510%
2023	49860.330	49905.304	44.9737	0.0902%
2024	50345.190	50163.650	181.5399	0.3606%
2025	50759.000	50395.783	363.2172	0.7156%

4. DISCUSSION

RK4 method demonstrates very consistent numerical stability across various step size variations in modeling logistic growth, although reducing the step size does not provide a significant improvement in accuracy. In comparison, the adaptive RKF45 method with a tolerance of 10^{-3} proved to be superior, as it was able to produce the smallest error rates with RMSE = 369.4707 thousand people and MAE = 273.9098 thousand people. The ability of RKF45 to dynamically adjust the step size based on local error estimates makes it more efficient and accurate compared to methods with a fixed step size. Overall, solving the logistic model using RKF45 is able to predict the population growth of West Java Province with excellent accuracy, where the relative error consistently remains below 2% for the entire observation period, averaging only about 0.5%.

Although proven to have high overall accuracy, this model is not without weaknesses when facing external shocks, as seen in the data anomaly in 2020 due to the COVID-19 pandemic. During that period, the model recorded a maximum error of up to 823.23 thousand people with relative error of 1.71% because the fundamental nature of the logistic model rigidly assumes a continuous growth pattern without accounting for extraordinary events. Despite this anomaly, this study confirms that the RKF45 method is highly recommended for projecting populations in other regions that have similar demographic characteristics. This method can be relied upon by policymakers, especially local governments, as an essential tool in developing sustainable development planning, such as providing infrastructure, educational facilities, and public health services.

5. CONCLUSIONS

Based on the results of the research and discussion described in the previous chapter, it can be concluded that the Runge-Kutta Fehlberg 45 (RKF45) method with a tolerance of 10^{-3} is the best method for predicting population growth in West Java Province. This method produced an RMSE of 369.4707 thousand people and an MAE of 273.9098 thousand people, which are smaller compared to the 4th Order Runge-Kutta (RK4) method, which produced an RMSE of 371.2286 thousand people and an MAE of 277.3376 thousand people.

RK4 method showed consistent results for various step size variations, indicating good numerical stability.

Meanwhile, the RKF45 method, with its adaptive capability, was able to produce smaller errors while being more efficient in terms of number of iterations and computation time. The data anomaly in 2020 due to the COVID-19 pandemic caused a significant increase in error, which is a limitation of the logistic model that assumes continuous growth. Overall, the relative prediction errors were below 2%, indicating that the logistic model solved with the RKF45 method has a good level of accuracy.

AI Disclosure Statement

During the preparation of this manuscript, the authors utilized DeepSeek and ChatGPT tools to assist in the development and writing of the programming code used for the numerical simulations of the RK4 and RKF45 methods. The authors have directly verified, thoroughly tested all generated code, and take full legal and ethical responsibility for the accuracy, integrity, and originality of the simulation results and the overall manuscript.

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