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Spontaneous Fission and Cluster Disintegration of Superheavy Nuclei

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ABSTRACT

The three parameters that determine that a super-heavy nucleus undergoes spontaneous fission (SF) or cluster disintegration (CD) are nuclear reaction energy, Q , half-life ($T_{1/2}$) of the super-heavy nucleus and the quantity $\frac{Z^2}{A}$ of the super-heavy nucleus. Using Modified Coulomb potential energy in the binding energy $B(A, Z)$ formula, values of Q , $T_{1/2}$, and $\frac{Z^2}{A}$ change. Calculating the magnitude of these quantities give the idea as to how the variations in N , Z and A determine the possibility of cluster disintegration and or spontaneous fission. Using Modified Coulomb energy formula in the expression for binding energy $B(A, Z)$ of the atomic nucleus, calculations are done for volume energy, surface energy, coulomb energy $B(A, Z)$ and nuclear reaction energy, Q . For the nucleus to disintegrate spontaneously, Q must be positive, and this is what is found in this article for the chosen super-heavy nuclei. Calculations show that Q is small for cluster disintegration and it is large for spontaneous fission. How the splitting of the core and neutrons in the neutron skin region are distributed among the fragments on disintegration is discussed, along with Alpha-particle structure of the nucleus.

Keywords: Super-heavy nuclei, Spontaneous fission, Cluster disintegration, Modified Coulomb energy.

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1. INTRODUCTION

Nuclear fission was discovered in 1938 and cluster radioactivity was discovered in 1984 [1]. There is still no fool proof theories to explain both phenomena, although a number of theories have been proposed from time to time. Induced nuclear fission and spontaneous nuclear fission are a special kind of nuclear reactions in which the excited compound nuclei disintegrate into two fragments of comparable masses and atomic numbers and some smaller particles like neutrons. Fission normally takes place among naturally occurring heaviest elements such as Uranium and Thorium. For heavy nuclei ($Z > 100$), there is spontaneous fission. Naturally occurring nuclei end at $Z = 92$; nuclei with $Z > 92$ are fabricated in the labs. Some of the super-heavy nuclei have very large half-life and they become part of the island of stability. There are many super-heavy nuclei that undergo spontaneous fission as soon as they are created in the lab, and their half-lives are of the order of 10^{-14} s to 10^{14} s.

The cluster radioactivity is the spontaneous emission of particles or clusters (of particles) such as Carbon ($^{12}_6C, ^{14}_6C$), oxygen ($^{16}_8O, ^{18}_8O$), Fluorine, Neon, Magnesium and Silicon *etc*, heavier than Alpha-particles ($^{20}_8O, ^{23}_9F, ^{22,24,26}_{10}Ne, ^{28-30}_{12}Mg, ^{34}_{14}Si$). It was discovered experimentally in 1984 [1], whereas it was proposed theoretically in 1980 [2]. These clusters were observed from nuclei $^{221}_{87}Fr$ to $^{242}_{96}Cm$ [3,4]. Extensive experimental work on cluster decay modes was done at Berkeley, Orsay, Dubna, and Milano. Some emitters were experimentally observed, and they included; and $^{242}_{96}Cm$. Similarly, the following cluster decays were observed, [5]



Decay of $^{223}_{89}Ac$ led to $^{15}_7N$; $^{226}_{90}Th$ gave $^{18}_8O$; $^{226}_{90}Th$ emitted $^{24,26}_{10}Ne$; $^{236}_{92}U$ emitted $^{24}_{10}Ne$; $^{232,233,235}_{92}U$ emitted $^{28}_{12}Mg$; $^{237}_{93}Np$ emitted $^{30}_{12}Mg$ and $^{240}_{94}Pu$ and $^{241}_{95}Am$ emitted $^{34}_{14}Si$. These are just some of the decay schemes; there could be many more decay modes, and up to now no odd-odd emitter has been observed. This could assist in understanding of cluster structures inside the atomic nuclei; as to whether they are formed just before emission or they get preformed as the value of A and Z reaches some critical value that results in disintegration. There is also a suggestion that in bound states, clustering is not an observable but rather an interpretation [6]. This suggests that clusters should pre-exist inside the nucleus before emission. There is no suggestion for preexisting fragments in the parent nucleus before fission. Moreover, there is no experimental evidence for signatures of cluster structures or fragments structures inside the atomic nucleus before disintegration. However, so far, the only definite existing knowledge is that competing Coulomb excitations (Coulomb deformation) and surface excitations (forces that tend to restore deformation) finally lead to disintegration. There still exists a gap between experimental observation and precision needed to validate many significant cluster theories proposed so far.

However, there is a recent study in which calculations have been done for super-heavy nuclei cluster-decay half-lives and their decay characteristics within the framework of the preformed cluster decay model (PCM), focusing on the doubly closed shell, $^{208}_{82}Pb$ daughter nucleus and some other daughter nuclei [8,9]. It is still not clear whether the clusters are formed just before integration or they already exist in the nucleus. Since the clusters and spontaneous fission fragments contain protons along with neutrons, they must come from the core of the nucleus that may be composed of equal number of protons and neutrons.

Then there are super-heavy nuclei that undergo both cluster disintegration and spontaneous fission. In this article, some such nuclei have been selected, and different forms of energy have been calculated in the binding energy formula $B(A, Z)$ and the value of Q .

Theory

The formula (for modified Coulomb energy) is written for the binding energy $B(A, Z)$ of the atomic nucleus in which the role played by modified Coulomb energy is included [9,10], i.e.,

$$B(A, Z) = a_1 A - a_2 A^{\frac{2}{3}} - \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R_0} e^{\frac{R_0^n}{nR^n}} - a_3 \frac{(A-2Z)^2}{A} \quad (2)$$

In Eq. (2), the first term is volume energy, second term is surface energy, third term is modified Coulomb energy, and fourth term is symmetry energy. Pairing energy has not been considered since super-nuclei have a core and neutron skin in which nucleons reside, where many body interactions are more relevant and strong. The parameters in Eq. (2) are:

$$a_1 = 15.85 \text{ MeV to } 15.85 \text{ MeV}$$

$$a_2 = 18.34 \text{ MeV (lie between } 13\text{-}18.5 \text{ MeV)}$$

$$a_3 = 23.21 \text{ MeV}$$

$$e = \text{electron charge} = 1.6 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12} \text{ F/m}$$

$$R_0 = \text{Radius of the core of the parent nucleus containing equal nuclear protons and neutron}$$

$$= 1.5 \times 10^{-15} \text{ m} \times (2Z)^{\frac{1}{3}}$$

$$R = \text{Radius of the parent nucleus} = R_0(A)^{\frac{1}{3}}$$

$$n = 0, 1, 2, 3, \dots, \infty$$

Using these parameters, energies in Eq. (2) are calculated to get the values of $B(A, Z)$ for different parent nuclei, the cluster decay nuclei and the fragments in the case of spontaneous fission.

Cluster radioactivity is a consequence of the competition between the restoring surface energy, $E_s = a_2 A^{\frac{2}{3}}$ and

the disruptive coulomb energy, $E_c = \frac{3}{5} \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 R_0} e^{\frac{R_0^n}{nR^n}}$.

The decay schemes are listed for some heavy and super-heavy nuclei, keeping in mind that decay products could be light-nuclei such as deuteron [11,12]; ${}^4_2\text{He}$, ${}^{12}_6\text{C}$, ${}^{18}_8\text{O}$, ${}^{24}_{10}\text{Ne}$, ${}^{28}_{12}\text{Mg}$, ${}^{34}_{14}\text{Si}$, ${}^{48}_{20}\text{Ca}$ and all these are even-even (even Z , even N) nuclei could be composed of Alpha-particle.

Cluster disintegration (CD) schemes are given for some of the heavy and super-heavy nuclei for which $B(A, Z)$ and Q are calculated ; Q is found to be positive.



The spontaneous fission (SF) decay schemes are listed for some super-heavy nuclei for which $B(A, Z)$ and Q are calculated; Q is found to be positive.



2. CALCULATIONS, RESULTS AND DISCUSSION

Using modified Coulomb energy formula in the expression for the binding energy, $B(A, Z)$ of the atomic nucleus, calculations are done for volume energy, surface energy, Coulomb energy (E_c), symmetry energy, binding energy $B(A, Z)$, and the nuclear reaction energy, Q . In Coulomb energy formula, since n can vary from $0 \rightarrow \infty$, calculations can be done for different values of n , but the whole calculation will be very lengthy. This article's purpose will be served by doing calculations for $n = 3$. There are two sets of calculations. One deals with cluster disintegration and the other deals with spontaneous fission (SF) of heavy and super-heavy nuclei. Many more such disintegration can be studied. Eqns. (3-8) are cluster decay schemes, and Eqns (9-18) are spontaneous fission schemes.

From the expression for Coulomb energy, it can be seen that $E_c \rightarrow \infty$ for $n = 0$; so this cannot be considered as valid result. For $n = \infty$, the Coulomb energy gets a multiplier of e (2.718).

In fact, for $n = 3$ onwards, the quantity $e^{\frac{R_o^n}{nR^n}} \rightarrow 1$, and hence the multiplier for Coulomb energy due to modified Coulomb energy will be 1.00. In such case, the Coulomb energy;

$$E_c = a_3 Z^2 \times \left(\frac{1}{A^{1/3}} \right) \quad (19)$$

Where;

$$a_3 = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R_o} = 0.71 \text{ MeV} \quad (20)$$

For $Z = 88$, $A = 222$, $n = 3$, it is found that:

$$E_c = 908.05 \text{ MeV} \quad (21)$$

This value is quite large and is due to the confinement of the Coulomb interaction within the nucleus, whereas Coulomb interaction is otherwise a long range interaction. It is well known that, for a nucleus of radius 3 fm and uniform spherical charge distribution, Coulomb energy component is 18.432 MeV . This is small compared to the value given in Eq. (21). As a rule, the magnitude of E_c should be large since it plays a key role in the disintegration of the nuclei. Since Coulomb energy is a repulsive energy, it reduces the overall binding energy of the nucleus. The greater the Coulomb repulsion, the lower the binding energy, thus promoting disintegration of the nucleus, leading to spontaneous fission (SF) for large Z in the case of super-heavy nuclei. For instance considering the reaction in Eq. (3), the nucleus $^{222}_{88}\text{Ra}$; Volume energy is 3516.7 MeV ; surface energy is 672.41 MeV ; Coulomb energy is 908.05 MeV as shown in **Table 1**.

For spontaneous disintegration, the magnitude of Q has to be finite and positive. Thus in this study, some heavy and super-heavy nuclei were chosen out of a large number of such nuclei that are available in literature, and calculated the Q values for some decay schemes **Table 1** gives values of Q for cluster decay, and **Table 2** provides values of Q for spontaneous fission of super-nuclei, in addition to the other 5 energies which play critical roles in the two decay schemes. The corresponding comparison of the various energies are displayed in **Figure 1** and **Figure 2** for the respective decay schemes.

Table 1 . Cluster Disintegration (CD) Scheme Energy Values.

Nucleus	A	Z	Z^2/A	Volume Energy (MeV)	Surface Energy (MeV)	Coulomb Energy (MeV)	Symmetry Energy (MeV)	Binding Energy, B (MeV)	Nuclear Reaction Energy, Q (MeV)
<i>Ra</i>	222	88	34.88	3516.7	672.41	908.05	221.22	2625.06	79.40
<i>C</i>	14	6	2.57	221.9	106.53	10.61	6.63	108.73	
<i>Pb</i>	208	82	32.33	3296.8	643.83	805.75	216.03	2436.93	
<i>Fm</i>	252	100	39.68	3994.2	731.70	1124.07	249.05	3013.45	142.54
<i>Si</i>	30	14	6.53	475.5	117.70	44.79	3.09	295.35	
<i>Rn</i>	220	86	33.63	3487.0	668.37	869.86	243.07	2575.56	
<i>No</i>	252	102	41.29	3994.2	731.70	1169.48	212.20	3050.30	146.88
<i>Ca</i>	48	20	8.33	760.8	242.23	78.15	30.94	487.62	
<i>Pb</i>	204	82	32.96	3233.4	635.56	810.98	182.04	2415.80	
<i>Fl</i>	284	114	45.76	4501.4	792.40	1403.77	256.29	3452.71	42.42
<i>He</i>	4	2	1.00	63.4	46.21	1.79	0	17.187	
<i>Cn</i>	280	112	44.80	4438.0	784.94	1361.37	259.95	3393.11	

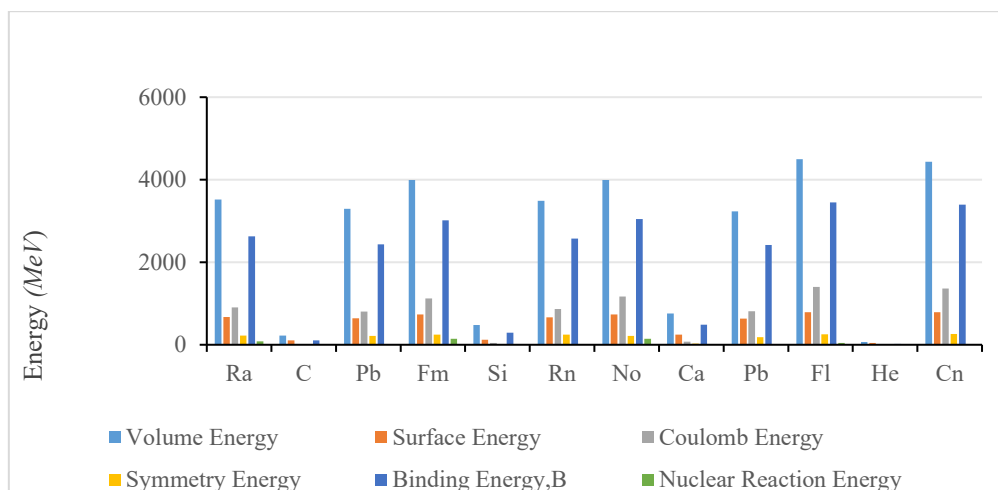


Figure 1. Comparison of Various Energies in Cluster Disintegration.

Table 2. Spontaneous Fission (SF) Scheme Energy Values.

Nucleus	A	Z	Z^2/A	Volume Energy (MeV)	Surface Energy (MeV)	Coulomb Energy (MeV)	Symmetry Energy (MeV)	Binding Energy, B (MeV)	Nuclear Reaction Energy, Q (MeV)
<i>Fl</i>	284	114	45.76	4501.40	792.40	1403.77	256.29	3452.71	210.13
<i>Xe</i>	139	54	20.98	2203.15	492.13	399.68	160.47	1550.56	
<i>Nd</i>	145	60	24.83	2298.25	506.19	486.52	100.04	1692.02	
<i>Fl</i>	284	114	45.76	4501.40	792.40	1403.77	256.29	3452.71	206.42
<i>Ba</i>	141	56	22.24	2234.85	496.85	427.79	138.44	1599.58	
<i>Ce</i>	143	58	23.52	2266.55	501.52	456.74	118.32	1646.71	
<i>Cn</i>	284	112	44.17	4501.40	792.40	1354.94	294.21	3414.79	205.45
<i>Te</i>	131	52	20.64	2076.35	473.06	378.01	129.16	1474.13	
<i>Nd</i>	153	60	23.53	2425.05	524.64	477.89	165.20	1735.21	

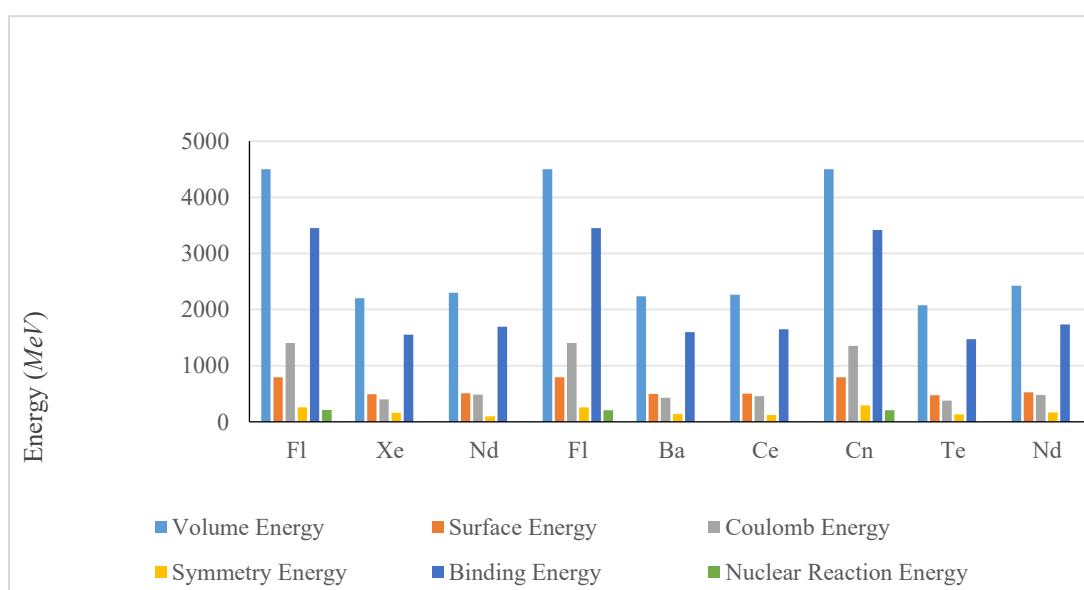


Figure 2. Comparison of Various Energies in Spontaneous Fission.

Nuclear reaction energy, Q , is the most important parameter to decide whether a super-heavy or heavy nucleus can disintegrate spontaneously. Another parameter to determine spontaneous disintegration is the half-life $T_{1/2}$ of the nucleus. Yet another parameter is the ratio $\frac{Z^2}{A}$. The most reliable parameter seems to be Q .

This is because it determines whether the decay is energetically allowed, provides the driving force towards a more stable configuration, and strongly controls the probability of barrier penetration. In heavy or super-heavy nuclei, even small changes in Q can translate to enormous differences in decay rates and half-lives, making it a primary quantity used to predict nuclear stability and decay behaviour. **Figure 3** displays nuclear reaction energy comparison of both CD and SF schemes.

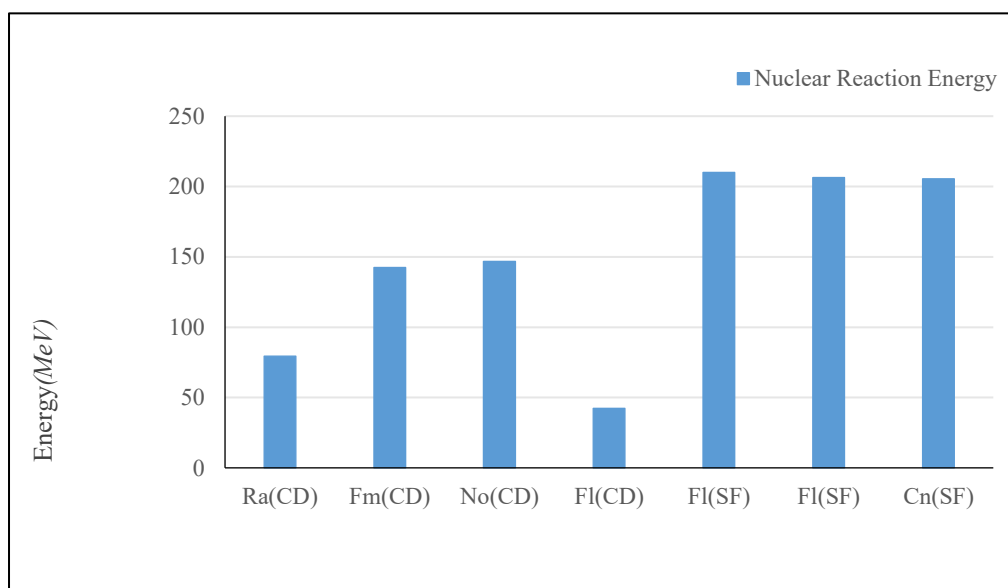


Figure 3. Comparison of Nuclear Reaction Energy Of Both.

In the cluster decay, the nuclei in *Eq. (3)* to *Eq. (8)* are even (Z -even)-even (N -even), and the fragments are also even-even. Also in the case of spontaneous fission (*SF*), the super-heavy nuclei are even-even, and the fragments are also even-even in the most cases, except that of *Eqns. (9), (10)* and *(17)*. How the fragments end up as even-even nuclei is still grey.

The discovery of radioactivity from atomic nuclei dates back to 1896 when it was found that nuclei spontaneously disintegrate by the emission of corpuscular radiations known as alpha-rays (α), electrons called β -rays and γ -rays in this order. The question as to why alpha-rays are emitted led to the study of atomic nuclei as composed of (α)-particles [13,14,15,16,17]. Some nuclei are even-even and could be assumed to be composed of (α)-particles. But as Z and A increase, number of neutrons N is much bigger than the number of protons ($N > Z$), and hence there will be excess neutrons, and this led to the symmetry energy term in $B(A, Z)$. Such a nucleus can thus be assumed to be composed of alpha-particles and neutrons, and excess neutrons may constitute the so-called skin region of the nucleus with thickness, say, n_s . In the atomic nucleus, the size of the neutron skin is sensitive to the strong interaction among protons and neutrons. The same interaction governs the behaviour of the matter that composes neutron stars [18]. In fact, in heavy nuclei in which the $N > Z$, neutrons may be nudged radially outward. At the edge of the nuclei, the neutrons exist in the skin- enclosing a core composed of equal number of neutrons and protons such that the core is composed of $2N = 2Z$ nucleons when $N = Z$.

In the case of $^{208}_{82}\text{Pb}$, which has 44 more neutrons than the proton number, the neutron skin turns out to be $(0.283 \pm 0.071)\text{fm}$ [19]. Whereas, if we calculate the neutron skin thickness (n_s) by assuming that the core of $^{208}_{82}\text{Pb}$ is composed of equal numbers of protons and neutrons ($2Z=164$), and the extra neutrons (44) stay in the skin region; and use the formula for the radius of the nucleus $R = R_o (1.2-1.5\text{fm}) \times A^{\frac{1}{3}}$, then the magnitude of the neutron skin turns out to be $n_s \cong 0.6769$ which is large compared to the experimental value [19]. The reduction in n_s could be due to the proton charge radius extension [10]. In fact, the neutron skin has been defined as the difference between the root-mean square (RMS) radius of the neutron distribution and the RMS radius of the proton distribution. Simultaneously, it is said that in the neutron-rich nuclei, the excess neutrons tend to occupy the outer region of the nucleus, forming a 'skin' around the core of protons and neutrons. Although the protons are confined to the core of the nucleus, their charge radius extends beyond the surface of the core envelop, and this effectively reduces the thickness of the neutron skin when measured experimentally. Also since symmetry energy in $B(A, Z)$ is a consequence of neutron excess, the neutron skin, n_s , should be sensitive to nuclear symmetry energy which describes how the energy of the nucleus or nuclear matter changes with the neutron-proton ratio. However, different theoretical models can predict significantly different neutron thickness for the same nucleus. Experimental uncertainties in measurement of neutron skin, n_s , are model dependent. Short life-times of superheavy nuclei inhibit exact measurement. Now, if the core of the parent nucleus is composed of Alpha-particles; then there are nuclei with odd- Z , and this can mean that there may exist a deuteron in the core, and this deuteron splits at a time of disintegration or is emitted. As such, study of neutron-deficient nuclei cluster radioactivity is of great importance [20,21,22].

3. CONCLUSION AND RECOMMENDATION

It is observed that spontaneous fission takes place when crossing over from some heavy nuclei to super-heavy nuclei. This happens when the Z value increases and the N value increases much more than Z . Since the neutrons account for large attractive energy, upon disintegration this is released, leading to large value of Q for spontaneous fission.

The values of Q for cluster disintegration are somewhat smaller compared to the Q values for spontaneous fission decay schemes, as shown in **Figure 3**. This is attributed to the fact that cluster decay involves only a partial restructuring of the nucleus, whereas in spontaneous fission the parent nucleus splits into two fragments of comparable size. The resulting fragments lie much closer to the region of maximum binding energy per nucleon. This produces a much larger gain in total binding energy.

Another important observation has been made in the sense that whether it is cluster decay or fission, the core of the parent nucleus splits to become the core of the fragments, and the excess neutrons in the neutron skin are distributed among the neutron skins of the fragments. How this happens is still an open question. A study on whether the alpha-particles exist permanently in the core of the nucleus, and a time of disintegration of the alpha-particles come together to form clusters in cluster radioactivity, is under consideration in the subsequent articles. Another study in future will be to ascertain as to why or how the core of the super-heavy nuclei splits to become the core of fragments. Also, how the excess neutrons in the neutron skin of the parent nucleus are distributed in the neutron skin of the fragments is open for investigation.

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