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The Specific Heat for Black Holes

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ABSTRACT

Specific heat (C_v) is an important thermodynamic parameter that has been used to study phase transition and to calculate the transition temperature (T_c) from one phase (that may be unstable) to another phase (that may be stable). The specific heat of a Black Hole could be negative or positive depending on its type and the thermodynamic variables kept constant. Negative C_v means that the Black Hole gets hotter as it radiates energy and this indicates instability and evaporation via Hawking radiation. Whereas in the case of larger Reissner-Nord-Strom and anti-de-Sitter (Ads) Black Holes, the specific heat can be positive, allowing them to achieve stable thermal equilibrium with their environment or surrounding atmosphere. Thus, the stability of the Black Hole is determined by the sign of the specific heat. With negative C_v, it may radiate energy and become smaller Black Hole, whereas with positive C_v, it may be large Black Hole with some phase transition. Calculations of C_v are done for two types of Black Holes. In one it is treated as a huge gravitational mass while in the second calculation, Black Hole is treated as a quantum gravitational mass. In both cases C_v is negative but in the quantum gravitational case C_v is less negative within intermediate temperatures. At higher temperatures of the order 10^{31}K both cases display a unified behavior signifying stability. Quantum gravity considerations lead to increase in T_c and this means a more stable system.

Keywords: Phase transition, Black Holes, Transition temperature, Negative Specific heat, Positive, Specific heat, Hawking Radiation

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INTRODUCTION

Uniquely, Black Holes inhabit a position in gravitational physics, marking regimes where space-time curvature step up to the point that causal disconnection becomes inescapable. These objects arise naturally from Einstein's field equations, yet for time out of mind they were understood primarily as geometric constructs rather than physical systems. Their characterization relied solely on a small set of conserved quantities—mass, electric charge, and angular momentum—leading to the prevailing assumption that Black Holes possessed no internal structure capable of supporting thermodynamic property such as temperature or entropy [1]. Within this classical outlook, Black Holes were thought out as inert endpoints of gravitational collapse rather than active participants in physical process. This viewpoint underwent a decisive transmutation with the recognition that Black Holes fulfill relations closely depicting the laws of thermodynamics. The association of horizon area with entropy and surface gravity with temperature revealed that Black Holes evolve in ways consistent with irreversible processes and energy equilibrium principles [2,3]. The theoretical prediction and subsequent interpretation of Hawking radiation resulted to the fact that Black Holes emit particles with a thermal spectrum, confirming that quantum effects provide them with a well-defined temperature [4]. These developments elevated Black Hole thermodynamics from a formal illation to a physically grounded framework, applicable to a wide range of solutions including rotating, charged, and cosmological Black Holes [5–7].

Although this framework is now well established, not all of its components are equally well founded at a fundamental level. The specific heat of a Black Hole, which governs its thermal response and stability, remains conceptually under-explored. Existing treatments typically obtain it through differentiation of temperature with respect to mass, offering limited insight into its deeper physical origin. While the appearance of negative specific heat is widely acknowledged and linked to instability, a principled derivation rooted in first principles is still lacking. This study seeks to address this deficiency by developing a systematic theoretical derivation of Black Hole specific heat, thereby clarifying its role within gravitational thermodynamics. The conceptual foundation of Black Hole thermodynamics emerged from the formulation of Black Hole mechanics, which uncovered precise relationships linking variations in mass, horizon area, angular momentum, and electric charge [2]. Initially, these relations were regarded as mathematical curiosities, since classical Black Holes were considered to absorb but never emit radiation. Bekenstein's proposal that horizon area measures entropy challenged this view by proposing that gravitational collapse stores information geometrically rather than dynamically [3]. This hypothesis acquired firm physical meaning when Hawking demonstrated that quantum fields near the horizon bring forth thermal radiation, assigning Black Holes a temperature determined by surface gravity [4].

Subsequent investigations confirmed that these thermodynamic attributes are not home-bound to the simplest Black Hole solutions. Rotating and charged Black Holes were shown to obey similar thermodynamic relations, reinforcing the universality of the framework [5,6]. Comprehensive reviews have since emphasized both the resilience of Black Hole thermodynamics across classical and semi-classical regimes and the unresolved issues surrounding statistical interpretation and steadiness [1,7,8]. The thermal behavior of Black Holes is extremely sensitive to the global structure of space-time. In asymptotically flat settings, Black Holes cannot maintain thermal equilibrium, as energy loss through radiation leads to progressive instability. By contrast, anti–de Sitter (AdS) space-times impose effective boundary conditions that allow equilibrium configurations. The discovery of the Hawking–Page transition demonstrated that Black Holes in AdS space can undergo phase changes correspondent to those found in conventional thermodynamic systems [9].

Further work disclosed that charged and rotating AdS Black Holes exhibit critical phenomena resembling those of interacting fluids, including first-order transitions and critical points [10]. The introduction of extended Black Hole thermodynamics, in which the cosmological constant is reinterpreted as pressure and mass as enthalpy, expanded this analogy and introduced new thermodynamic variables [11]. Within this framework, specific heat functions as a key indicator of local stability, yet its treatment remains largely computational rather than foundational. Research has also explored Black Hole thermodynamics in gravitational theories that go beyond general relativity. In adapted frameworks such as f(R) gravity and massive gravity, deviations from the standard entropy–area relation alter temperature and stability conditions, resulting in modified heat capacities [12,13]. Quantum corrections to entropy, including logarithmic and non-linear contributions, further modify thermodynamic response functions and can shift stability regimes [14,15]. Simplified models, such as lower-dimensional Black Holes and semiclassical approximations, provide controlled settings for examining quantum effects on thermodynamic quantities [16,17]. While these studies enrich the phenomenology of Black Hole thermodynamics, they typically treat specific heat as a quantity inferred from pre-existing relations rather than derived from underlying principles.

Attempts to explicate Black Hole thermodynamics from a microscopic standpoint have produced several influential approaches. In string theory, explicit microstate counting has reproduced Black Hole entropy for specific configurations, lending support to a statistical interpretation of gravitational thermodynamics [18]. Holographic duality further links Black Hole behaviour to thermal properties of strongly couple quantum field theories, offering indirect insight into response functions such as heat capacity [19].

Alternative statistical formalisms, including none-extensive and generalized entropy frameworks, have also been applied to Black Holes, yielding modified thermodynamic relations and stability criteria [20,21]. Despite their conceptual diverseness, these approaches by and large assume known temperature–entropy relationships, leaving the origin of specific heat insufficiently addressed. Across classical, extended, and quantum-corrected settings, the literature consistently treats Black Hole specific heat as a secondary construct rather than a primary thermodynamic quantity. Although its negative sign and associated instability have been extensively documented [22–24], a unified derivation rooted directly in first principles is still nonexistent. This unresolved issue arouse interest in the present work, which aims to establish a systematic theoretical derivation of Black Hole specific heat and to enhance the conceptual wholeness of Black Hole thermodynamics.

THEORETICAL DERIVATIONS

For Black Holes, the specific heat is calculated from the equation;

$$C = T\left(\frac{dS}{dT}\right) \tag{1}$$

Where T = Black Hole's Hawking temperature and S = Entropy. This definition shows how the Black Hole's temperature changes in response to the added heat, while holding other variables constant.

The Black Hole's temperature formula is,

$$T = \frac{\hbar c^3}{8\pi kGM} \tag{2}$$

Where:

k = Boltzmann Constant $= 1.38\times10^{-23} JK^{-1}$

c = Speed of light $= 3\times10^{8} ms^{-1}$

$\hbar$ = Reduced Planck's constant $= \frac{1}{2\pi}\times 6.626\times10^{-34} Js$

G = Gravitational constant $= 6.67\times10^{-11} m^3 kg^{-1} s^{-2}$

M = Mass of the Black Hole

Now we can write, energy E of the Black Hole as,

$$E = Mc^2 \text{ or } M = \frac{E}{c^2} \tag{3}$$

Substituting for M in Eq. (2) from Eq. (3), yields,

$$T = \frac{\hbar c^3 c^2}{8\pi kGE} \tag{4}$$

Eq. (4) shows that E is a function of temperature T or,

$$E = E(T) \tag{5}$$

Thus,

$$E = \frac{\hbar c^5}{8\pi kGT} \tag{6}$$

Specific heat C_v is written as,

$$C_v = \left(\frac{\partial E}{\partial T}\right)_v \tag{7}$$

From Eq. (7) the specific heat of the Black Hole is calculated is determined by,

$$C_v = -\frac{\hbar c^5}{8\pi kGT^2} \tag{8}$$

This shows that the specific heat is negative. The question arises whether this will be true for rotating and charged Black Holes? However, it is a fact that the specific heat of Black Holes is negative as this fact is true for self-gravitating systems such as stars.

For rotating and charged Black Hole, there are some intricate changes. The energy E is still given by $E = Mc^2$, but the general expression for the Black Hole temperature is different; that is,

$$T = \frac{\kappa\hbar}{2\pi kc} \tag{9}$$

Where:
κ = Black Hole surface gravity, and for Schwarzschild Black Hole, it is given by

$$\kappa = \frac{c^4}{4MG} \tag{10}$$

Substituting for κ from Eq. (10) in Eq. (9) gives,

$$T = \frac{\hbar}{2\pi kc} \cdot \frac{c^4}{4MG} = \frac{\hbar c^3}{8\pi kMG} \tag{11}$$

Eq. (4) and Eq. (11) are identical equations for the temperature T of the Black Hole.
For a charged, rotating Black Hole, known as, kerr-Newman Black Hole, the value of κ is,

$$\kappa = \frac{\left(M^2 - a^2 - Q^2\right)^{\frac{1}{2}}}{2M\left(M + \sqrt{M^2 - a^2 - Q^2}\right) - Q^2} \tag{12}$$

Where Q = charge on the Black Hole, and the angular momentum J= a.M (thermodynamics from total energy of a Black Hole and its temperature)

In Eq. (11) using $E = Mc^2$, yields,

$$T = \frac{\hbar c^3 c^2}{8\pi kGE} = \frac{\hbar c^5}{8\pi kGE} \tag{13}$$

And thus,

$$E(T) = \frac{\hbar c^5}{8\pi kGT} \tag{14}$$

$$C_v = \frac{\partial E(T)}{\partial T} = -\frac{\hbar c^5}{8\pi kGT^2} \tag{15}$$

Which is the same as Eq. (8).
Now due to quantum gravity in which the macroscopic gravitational field is unified with microscopic quantum mechanics; a many body quantum gravity system can be found such as a Black Hole. In that case *E(T)* is multiplied by quantum many body thermal activation factor, also called Boltzmann statistical function $e^{-\frac{E(T)}{kT}}$ to get a new expression for energy, say

$$u(T) = E(T)e^{-\frac{E(T)}{kT}} \tag{16}$$

Eq. (16) is now used to calculate C_v as follows,

$$C_v = \frac{\partial u(T)}{\partial T} = \frac{\partial}{\partial T}\left[E(T)e^{-\frac{E(T)}{kT}}\right]$$

$$C_v = E(T)\frac{\partial}{\partial T}\left[e^{-\frac{E(T)}{kT}}\right] + e^{-\frac{E(T)}{kT}}\frac{\partial}{\partial T}[E(T)]$$

$$C_v = E(T)\frac{\partial}{\partial T}\left[e^{-\frac{E(T)}{kT}}\right]\left[\frac{\partial}{\partial T}\left\{\frac{E(T)}{kT}\right\}\right] + e^{-\frac{E(T)}{kT}}\frac{\partial}{\partial T}[E(T)] \quad (17)$$

Now,

$$\frac{E(T)}{kT} = \frac{\hbar c^5}{8\pi k^2 G T^2} \quad (18)$$

$$\frac{\partial}{\partial T}[E(T)] = -\frac{\hbar c^5}{8\pi k G T^2} \quad (19)$$

$$\frac{\partial}{\partial T}\left[\frac{E(T)}{kT}\right] = \frac{\partial}{\partial T}\left[\frac{\hbar c^5}{8\pi k^2 G T^2}\right]$$

$$\frac{\partial}{\partial T}\left[\frac{E(T)}{kT}\right] = -\frac{2\hbar c^5}{8\pi k^2 G T^3} \quad (20)$$

Substituting from Eq. (19) and Eq. (20) into Eq. (17) gives C_v for quantum gravity many-body system (Black Hole)

$$C_v = \left(e^{-\frac{E(T)}{kT}}\right)\left\{\left[E(T)\left(-\frac{2\hbar c^5}{8\pi k^2 G T^3}\right)\right] + \left(-\frac{\hbar c^5}{8\pi k G T^2}\right)\right\} \quad (21)$$

$$C_v = \left(e^{-\frac{\hbar c^5}{8\pi k^2 G T^2}}\right)\left[\left(-\frac{2\hbar^2 c^{10}}{64\pi^2 k^3 G^2 T^4}\right) - \left(\frac{\hbar c^5}{8\pi k G T^2}\right)\right] \quad (22)$$

Let $\frac{\hbar c^5}{8\pi k^2 G} = \alpha$ (23)

$$C_v = \left(e^{-\frac{\alpha}{T^2}}\right)\left[\left(-\frac{2\hbar c^5 \alpha}{8\pi k G T^4}\right) - \left(\frac{k\alpha}{T^2}\right)\right]$$

$$C_v = -\left(e^{-\frac{\alpha}{T^2}}\right)\left[\left(\frac{2\hbar c^5 \alpha}{8\pi k G T^4}\right) + \left(\frac{k\alpha}{T^2}\right)\right] \quad (24)$$

The specific heat is negative showing that even the quantum gravity many-body system has negative specific heat that when the Black Hole loses heat (energy) its temperature increases or it becomes more unstable. Only the temperature variation and magnitude has changed. Eq. (24) can be finally written as,

$$C_v = -\left(e^{-\frac{\alpha}{T^2}}\right)\left[\left(\frac{2k\alpha^2}{T^4}\right) + \left(\frac{k\alpha}{T^2}\right)\right] \quad (25)$$

$$C_v = -\left(\frac{k\alpha}{T^2}\right)\left(e^{-\frac{\alpha}{T^2}}\right) - \left(\frac{2k\alpha^2}{T^4}\right)\left(e^{-\frac{\alpha}{T^2}}\right) \tag{26}$$

If,

$$\left(\frac{\alpha}{T^2}\right) = 1, \tag{27}$$

Then,

$$T = \sqrt{\alpha}$$
$$T = \sqrt{7.429\times 10^{62}}$$
$$T = 2.726\times 10^{31} K \tag{28}$$

And so,

$$C_v = -(k)\left(\frac{1}{e}\right) - (2k)\left(\frac{1}{e}\right) = \left(\frac{3k}{e}\right) = -1.10375\times 1.38\times 10^{-23} = 1.523175\times 10^{-23} JK^{-1}mol^{-1} \tag{29}$$

Thus, C_v is negative and constant.

Therefore, to study the variation of C_v against T, the exact form of Eq. (26) has to be used. The exponential and the second terms are a consequence of considering the systems as a quantum gravity (system) many body system, and its temperature effects are taken into account via quantum many body thermal activation factor $e^{-\frac{E}{kT}}$. Eq. (15) and Eq. (26) for C_v can be used to calculate the transition temperature Tc for the ordinary system and quantum gravity many-body system, i.e., $\left(\frac{\partial C_v}{\partial T}\right)_{T=T_c} = 0$, determines Tc. Eq. (15) leads to infinite value for T_c (very large), whereas Eq. (26) gives finite but large value which is physically more justified or more acceptable thermodynamically.

RESULTS AND DISCUSSION

The presence of negative specific heat as represented by Eq. (15) and Eq. (26) and displayed by the graphs *Cv1* and *Cv2* respectively as shown in Fig. 1 reveals an essential aspect of Black Hole physics.

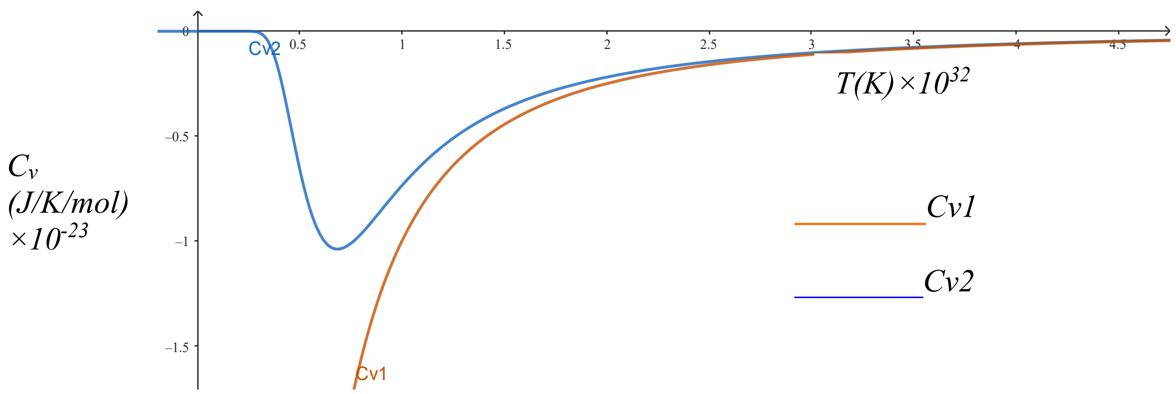

Figure 1. Graph of Cv against T.

As temperature increases slightly C_v decreases sharply and becomes negative. This indicates an anomalous thermodynamic regime, typical of systems with long-range interaction. The minimum corresponds to a region of maximum thermodynamic instability. Within the intermediate temperatures, both curves increase monotonically with temperature. The magnitude of the negative heat capacity decreases. Cv_2 remains less negative than Cv_1 over this range. As T becomes large both curves approach zero from below. The two curves converge, indicating that the difference between Cv_1 and Cv_2 becomes negligible. This suggests a transition toward a quasi-stable or classical regime, where the system behaves more conventionally.

As energy is carried away by Hawking radiation, the Black Hole does not cool in the usual thermodynamic sense; instead, its temperature rises as its mass decreases. This inverse response produces an unstable feedback process in which energy loss accelerates evaporation rather than driving the system toward equilibrium. Such behavior lies outside the assumptions underlying canonical thermodynamic descriptions, clarifying why an isolated Black Hole in asymptotically flat space-time cannot maintain a stable thermal balance. Far from signaling an imperfection in the theory, this outcome indicates the intrinsically gravitational character of Black Holes, where long-range interactions fundamentally alter the applicability of ordinary thermodynamic reasoning.

Talking of a Schwarzschild Black Hole, the temperature varies inversely with mass, $T \propto \frac{1}{M}$, thus when the Black Hole acquires energy its temperature decreases, and when it loses energy its temperature increases. Given that the specific heat is defined as $C_v = \frac{dE}{dT}$, this inverse relationship results to C_v being negative. What is more, $C_v \propto -\frac{1}{T^2}$, and so as T^2 increases the size of C_v decreases, causing the curve to flatten and asymptotically approach zero from below. This negative specific heat reflects the thermodynamic imbalance characteristic of self-gravitating systems such as Black Holes.

The calculations in this manuscript lead to C_v being negative and transition temperature Tc depends on whether the Black Hole is considered as a huge gravitational mass with event horizon as environment or as a quantum gravitational huge mass. Both become stable and do not radiate any energy. Recent observations on the emissions of massive jet of hot plasma that streams from the center of the Black Hole in the center of the galaxy known as M87 by Professor Jan Roder at the institute of Astro-Physics Andabusia, Spain, confirms that the Black Hole may radiate energy. This matter will be taken up in the next manuscript on entropy of Black Holes.

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