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Robust Analysis of Internal Rate of Return under Cash Flow Volatility using the Proposed Optimal Numerical Method (PONM)

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ABSTRACT

This study analyzes the robustness and efficiency of Internal Rate of Return (IRR) calculations under stochastically fluctuating cash flow conditions. Three numerical methods are compared: Newton–Raphson, Secant, and the Proposed Optimal Numerical Method (PONM). Empirical datasets and lognormal stochastic simulations are used to test algorithmic performance against volatility and random disturbances. Experimental results show that PONM achieves the fastest convergence, with an average of 5.98 iterations, a success ratio of 99.5%, and the smallest deviation $\sigma = 0.0095$. Robustness and sensitivity tests show that PONM has the lowest Coefficient of Variation and Shock Sensitivity, indicating the highest numerical stability. With a fourth-order convergence, PONM proves more efficient and robust to noise than classical methods. These findings confirm the relevance of PONM as an optimal algorithm for IRR calculations in a highly volatile investment environment.

***Keywords*:** Internal Rate of Return, Cash Flow Volatility, Stochastic Simulation, Root-Finding Method, Proposed Optimal Numerical Method.

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1. INTRODUCTION

The concept of the Internal Rate of Return (IRR) has long been a key pillar in investment evaluation and capital budgeting theory. Historically, the idea of IRR emerged from classical economic debates rooted in the idea of [1], [2] and [3] which discusses the relationship between capital efficiency and a "fair" rate of return on investment. In the modern framework, IRR is understood as the internal rate of return that equalizes the present value of a project's cash inflows and outflows, thus reflecting the balance between investment and future financial benefits [4]. This concept has become a universal measure in assessing project profitability because it is intuitive, easy to interpret, and can be directly compared with other benchmarks such as return on investment (ROI) or cost of capital [5]. The Internal Rate of Return (IRR) is a central metric in the assessment of investment projects. It is widely used in corporate and investment practice as a measure of relative returns; however, it has several conceptual and numerical limitations that are important for application to real cash flows [6].

In modern finance, project cash flows often exhibit high volatility, with large fluctuations, unconventional signs, and the presence of noise [7]. This situation creates two critical problems for the IRR: first, the emergence of more than one root (multiple IRRs) or the absence of a single solution to the net present value equation; second, extreme sensitivity, where the IRR value changes sharply due to small changes in cash flow components, making interpretation and decision-making difficul [8]. Modern finance literature emphasizes that the multiple IRR phenomenon is not simply a numerical anomaly, but rather a structural problem that requires a more robust conceptual and mathematical approach [9].

IRR calculations rely on numerical root finding methods such as Newton-Raphson, secant, and their variations [10]. However, these classical methods often fail to converge or produce unstable results when the present value function has a small gradient, multiple roots, or initial guesses far from the true roots. Several numerical engineering studies have proposed Newton variants and intelligent initialization methods, but efficiency and stability remain major challenges [11]. Therefore, a more robust, efficient numerical approach is needed that is capable of handling volatile cash flow conditions [12]. Moreover, recent numerical finance research demonstrates that cash-flow patterns with multiple sign reversals or irregular timing significantly increase the likelihood of non-convergence and spurious IRR solutions, reinforcing the need for numerical schemes that are less sensitive to input volatility [13]. In addition, comparative numerical investigations have shown that classical Newton-based algorithms often converge to incorrect roots or diverge entirely when the NPV function is highly non-linear due to alternating or unevenly distributed cash-flow patterns, and that hybrid approaches incorporating bracketing have produced more stable IRR estimates under such volatility [14].

There is a gap at the intersection of three research domains that have been running separately, namely: (a) capital budgeting theory that discusses the characteristics of IRR and the problem of multiple IRRs, (b) sensitivity analysis and the resilience of IRR to cash flow volatility, and (c) the development of optimal numerical methods to determine IRR efficiently and stably. Most previous studies have only highlighted one aspect separately, so there is still little research that combines all three in an integrated manner, namely, examining the behavior of IRR under cash flow volatility while simultaneously testing the effectiveness of higher-order numerical methods such as Proposed Optimal Numerical Method (PONM) under these conditions.

This research aims to design and test a robust numerical-analytical framework for IRR under cash flow volatility, using the PONM, that is a fourth-order root-finding method that requires only the evaluation of functions and their derivatives and has robust local and semilocal convergence analysis. PONM offers a balance between computational efficiency and robustness to adverse initial conditions, making it particularly relevant for sensitive IRR problems. Mathematically, PONM is developed from a combination of the Chebyshev–Halley and Potra–Pták methods and satisfies the optimality constraints according to the Kung–Traub criterion [15].

This study comprehensively analyzes the effect of cash flow volatility on the mathematical behavior of IRR and the effectiveness of the numerical methods used in its determination. Analytically, this study examines how the characteristics of cash flow volatility—such as variance, autocorrelation, and shock scenarios. This will affect the root structure of the net present value equation, including the number of real roots, the closeness between the roots, and their local sensitivity. Numerically, PONM is applied to calculate IRR under various volatile cash flow scenarios, then compared with classical methods such as Newton and Halley based on iteration efficiency, radius of convergence, and robustness to stochastic disturbances.

Therefore, the findings of this study deliver three integrated forms of contribution. First, a methodological contribution in the form of adopting and validating the Proposed Optimal Numerical Method (PONM) as an efficient algorithm for IRR computation under volatile cash flow conditions. Second, a theoretical contribution by establishing a mathematical characterization of IRR sensitivity with respect to stochastic fluctuations in cash flows—linking NPV root geometry, variance dynamics, and convergence behavior of numerical algorithms. Third, an applicative contribution, by formulating practical investment guidelines that inform investors on how cash flow volatility and numerical robustness influence the reliability of IRR-based decision-making.

2. MATERIAL AND METHODS

2.1. Research Design and Data Setup

This study adopts a quantitative–computational design to estimate the Internal Rate of Return (IRR) under conditions of stochastically fluctuating cash flows. Conceptually, IRR is defined as the root of the Net Present Value (NPV) function:

$$f(r) = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} = 0, \qquad (1)$$

where CF_t denotes the cash flow at period t, and r represents the internal rate of return. To obtain the roots of the NPV equation efficiently and stably, this study employs the Proposed Optimal Numerical Method (PONM)—a fourth-order iterative method requiring only two function evaluations and one derivative per iteration, and satisfying the Kung–Traub optimality conjecture. PONM is compared with two classical algorithms—Newton–Raphson and Secant—to assess robustness, convergence speed, and numerical stability under cash flow volatility.

The data used in this research consists of two complementary components: synthetic and empirical datasets. Synthetic data enables controlled simulation of market conditions with varying levels of volatility, while empirical data serves as a real-market benchmark to validate the simulation outcomes. The synthetic dataset is generated through stochastic simulation, whereas the empirical dataset is taken from the corporate and financial sectors.

Synthetic cash flows are generated based on three probabilistic models to represent diverse real-world financial dynamics: (i) a lognormal distribution to characterize conventionally positive cash flows with right-skewed volatility, (ii) a Student-(t) distribution to model extreme fluctuations and heavy-tailed behavior, and (iii) a GARCH(1,1) model to capture clustered volatility over time. These simulations incorporate empirical parameter calibration derived from real data and are executed over a one-year horizon ($t = 0,1,\dots,10$). Three volatility regimes are applied—low, moderate, and high—by scaling the volatility parameter σ to 0.5σ, σ, and 1.5σ. In total, each model–volatility combination generates 10,000 Monte Carlo scenarios, producing a dataset suitable for rigorous numerical stress-testing.

To ensure empirical validity, the simulation setup is calibrated using real investment project cash flow data obtained from the Bloomberg Corporate Bonds (Baa-rated) and FRED Economic Data for the 2020–2025 period. The empirical dataset shows a smooth positive trend with moderate volatility, no sign changes (indicating conventional cash flows), and strong inter-temporal dependence, evidenced by a high autocorrelation coefficient. All experiments were run on a Python 3.11 computing system with the NumPy, SciPy, and Matplotlib libraries. Statistical cleaning and preprocessing yield the key parameters $\mu = 4532.91$ and $\sigma = 982.18$, which are subsequently transformed into lognormal parameters $\mu_{log} = 8.396$ and $\sigma_{log} = 0.214$. These calibrated values are then embedded into the Monte Carlo simulation to produce synthetic cash flows that closely mimic real financial patterns.

The combined use of a numerical–analytical framework using PONM and realistic cash flow scenarios using calibrated stochastic models enables this research to systematically evaluate the robustness, efficiency, and stability of IRR calculations against volatility, random disturbances, and structural uncertainty. This integrated design provides a rigorous foundation for the subsequent computational experiments and robustness assessments.

2.2. Convergence of the PONM method

The PONM algorithm implementation was adapted from the formulation of [15]. The PONM method was implemented to solve nonlinear equations, such as that in (1),

$$f(r) = 0,$$

which in an economic context usually represents the NPV (Net Present Value) function against the interest rate r. The goal is to find simple roots r^* that satisfy $f(r^*) = 0$.

PONM is a two-step iterative method obtained from the afine combination of two third-order convergence methods, namely the Zhou Method, a modified version of Chebyshev–Halley and the Potra–Pták method (1984). Through a linear combination with parameters, the general form is obtained:

$$x_{n+1} = x_n - \eta \frac{f(x_n) + f(y_n)}{f'(x_n)} - (1-\eta) \frac{f(x_n)^2 - 2f(x_n)f(y_n)}{f'(x_n)[f(x_n) - 3f(y_n)]},$$

with

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \quad \eta \in R.$$

The parameter $\eta = \frac{1}{3}$ selection results in a fourth-order optimal method according to the Kung–Traub Conjecture (1974), where $\kappa = 3$ for the function evaluation per iteration; the theoretical maximum order is $2^{\kappa-1} = 4$. With $\eta = \frac{1}{3}$, the explicit form of the PONM algorithm is

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2}$$

$$x_{n+1} = x_n - \frac{1}{3}\left[\frac{f(x_n) + f(y_n)}{f'(x_n)} - \frac{f(x_n)^2 - 2f(x_n)f(y_n)}{f'(x_n)\big(f(x_n) - 3f(y_n)\big)}\right]. \tag{3}$$

This method requires only two function evaluations and one first derivative evaluation per iteration, making it computationally efficient. Suppose the actual roots r^* satisfy $f(r^*) = 0$.
Defined:

$$e_n = x_n - r^* \quad \text{and} \quad d_n = y_n - r^*.$$

By expanding $f(x_n)$ and $f'(x_n)$ around r^* using Taylor series, we have:

$$f(x_n) = f'(r^*)\big(e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + O(e_n^5)\big) \tag{4}$$

$$f'(x_n) = f'(r^*)\big(1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + O(e_n^4)\big) \tag{5}$$

with

$$c_j = \frac{f^{(j)}(r^*)}{j!\, f'(r^*)}, \quad j = 2,3,4 \tag{6}$$

By dividing Equations (4) by (5), the following result is obtained:

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2 e_n^2 + 2(c_3 - c_2^2)e_n^3 + (7c_2c_3 - 4c_3^2 - 3c_4)e_n^4 + O(e_n^5). \tag{7}$$

Thus,

$$d_n = e_n - \frac{f(x_n)}{f'(x_n)} = c_2 e_n^2 + 2(c_3 - c_2^2)e_n^3 + (4c_2^3 + 3c_4 - 7c_2c_3)e_n^4 + O(e_n^5). \tag{8}$$

By expanding $f(y_n)$ around r^* using Taylor expansion, we have:

$$f(y_n) = f'(r^*)\big(d_n + c_2 d_n^2 + c_3 d_n^3 + c_4 d_n^4 + O(d_n^5)\big). \tag{9}$$

Substitution d_n to (9) yields:

$$f(y_n) = f'(r^*)\left(c_2 e_n^2 + 2(c_3 - c_2^2)e_n^3 + (5c_2^3 + 3c_4 - 7c_2c_3)e_n^4 + O(e_n^5)\right). \qquad (10)$$

Substituting all expansions (4), (5), and (9) into the PONM formula (10) gives the error equation:

$$e_{n+1} = -c_2(c_2^2 + c_3)e_n^4 + O(e_n^5) \qquad (11)$$

From the error in Equation (11):

$$|e_{n+1}| = K|e_n|^4 + O(|e_n|^5), \qquad (12)$$

with $K = |c_2(c_2^2 + c_3)|$ is an asymptotic constant. Therefore, it can be concluded that PONM has fourth-order convergence. This method also meets the optimal limit of Kung–Traub (1974) because it uses 3 evaluations per iteration and reaches order 4, namely:

$$p_{\max} = 2^{\kappa-1} = 2^{3-1} = 4.$$

The PONM iteration is stopped if:

$$|f(x_n)| < 10^{-10}.$$

For financial applications (e.g. finding the IRR), it is usually sufficient to use a tolerance value of 10^{-8} or 10^{-10}. PONM is faster than the Newton–Raphson method (2nd order) and the Chebyshev–Halley method (3rd order). The error decreases proportionally to $e_{n+1} \approx Ce_n^4$, indicating a significant acceleration in convergence because it only requires the first derivative $f'(x_n)$, this method remains efficient and practical for application to complex NPV functions.

2.3. Robustness and Sensitivity Analysis

To assess the robustness of the method to cash flow fluctuations, three main experiments were conducted:

1. Volatility Regime Test — tests the stability of IRR convergence at three levels of volatility: low $\sigma = 0.1$, medium $\sigma = 0.3$, and high $\sigma = 0.5$.
2. Outlier Stress Test — introduces extreme shocks to 5–10% of observations CF_t to test the breakdown point of the IRR estimator.
3. Convergence Efficiency Test — compares the number of iterations, computation time, and success rate of PONM with the Newton–Raphson and Secant methods.

In addition, a sensitivity analysis of local derivatives was performed using the Implicit Function Theorem (IFT):

$$\frac{\partial r}{\partial CF_t} = -\frac{1}{f'(r)}\frac{1}{(1+r)^t},$$

to measure the marginal effect of changes in cash flows on shifts in IRR δr under stochastic disturbances.

2.4. Performance Evaluation

This subsection evaluates the performance of the proposed PONM method based on accuracy, robustness, and computational efficiency. Accuracy is assessed through the deviation of the computed IRR from a high-precision numerical reference, robustness is measured by the stability of IRR estimates under data variation and outliers, and efficiency is evaluated using iteration counts and computation time to convergence. These criteria jointly provide a concise yet comprehensive assessment of the method's numerical reliability. Performance evaluation is done based on three main metrics:

(I) Accuracy — deviation of the PONM IRR result compared to the exact root (or numerical result with tolerance 10^{-12};

(II) Robustness — the level of stability of the IRR results against variations and the presence of outliers, measured by the coefficient of variation and influence function CF_t

(III) Efficiency — the number of iterations and the average computation time to reach convergence.

3. DATA AND EXPERIMENTS

This section presents the data sources and experimental design employed in this study, along with the procedures used to ensure data consistency and suitability for stochastic modeling. The empirical analysis is structured to bridge real-world financial observations with the theoretical framework developed in subsequent sections, ensuring that the calibrated parameters reflect actual market behavior. By combining high-frequency financial time series with rigorous preprocessing and validation steps, this experimental setup provides a reliable foundation for both descriptive analysis and simulation-based inference, thereby enhancing the robustness and interpretability of the proposed model.

3.1. Empirical Data and Parameter Calibration

Empirical datasets were downloaded from the Federal Reserve Economic Data (FRED) and the Bloomberg Corporate Bond Index (Baa-rated) for the period October 2020–October 2025. The first stage of this research aims to conduct descriptive analysis and statistical parameter calibration based on empirical data obtained from a daily time series of closing price indexes totaling 1,474 observations. This dataset represents the dynamics of cash flows or market prices, which are used as the basis for building a stochastic model in the subsequent simulation stage.

The empirical data was cleaned of non-numerical and missing values, then converted into a time series with period numbering ($t = 0, 1, 2, \ldots, 1473$). All values were positive, so the cash flow pattern was categorized as conventional cash flow — meaning there was only one cash outflow followed by a series of cash inflows (indicated by the resulting sign changes). This implies the existence of a single IRR root in the numerical analysis stage.

Statistically, the results of empirical data processing provide the following summary:

Table 1. The results of empirical data processing

Parameter	Mark	Interpretation
Number of observations (n)	1,474	The number of daily data points used.
Average (μ)	4,532.91	Average value of cash flow/price per period.
Standard deviation (σ')	982.18	The volatility measure is around the average.21,7%
Autocorrelation lag-1 (ρ_1)	0.9986	The relationship between periods is very strong (time-dependent).
Skewness	0.405	The distribution is slightly skewed to the right (positive).
Curtosis	−0.491	The distribution is flatter than normal (platykurtic).
Sign changes	0	All values are positive (conventional cash flows).

These results indicate that the empirical data exhibits a very smooth positive trend with moderate volatility and very high temporal dependence. The autocorrelation value is close to 1, indicating that interperiod fluctuations are very small, as is typical for smooth, continuous financial series.

Mathematically, empirical parameters are used to estimate the equivalent lognormal distribution parameters that will be used in the next stage of the stochastic simulation. Based on the estimation results, the following is obtained:

$$\mu_{log} = 8.396 \quad \text{and} \quad \sigma_{log} = 0.214$$

which is calculated using the transformation:

$$\sigma_{log}^2 = \ln\left(1 + \frac{\sigma^2}{\mu^2}\right),$$

$$u_{log} = \ln(\mu) - \frac{1}{2}\sigma_{log}^2.$$

These parameters $(\mu_{log}, \sigma_{log})$ represent the mean and dispersion on a logarithmic scale, probabilistically defining the empirical lognormal distribution of the data. This distribution is considered the most appropriate because all cash flow values are positive and tend to fluctuate around the mean with relatively small deviations.

From these results, it can be concluded that the empirical data indicates high financial stability and a lognormal distribution structure with moderate volatility. Therefore, the empirical parameters, $\mu = 4532.91$, $\sigma = 982.18$, $\mu_{log} = 8.396$, and $\sigma_{log} = 0.214$ will be used as the basic parameters in generating synthetic data in next stage on Section 3.2 of the experiment, namely Monte Carlo simulation with three levels of volatility (low, medium, and high).

3.2. Stochastic Simulation of Cash Flows

The second stage of this research aims to generate synthetic cash flow data that resembles the empirical characteristics of the results from Section 3.1. This process is carried out through Monte Carlo-based stochastic simulations using three probability distribution models, namely Lognormal, Student-t, and GARCH(1,1).

These models were chosen to represent three types of dynamics commonly encountered in financial data:

(I) The Lognormal model is used for positive cash flows with asymmetric distribution to the right, according to the empirical results from Section 3.1 which show all values ($CF_t > 0$).
(II) The Student-t model is used to describe the heavy-tail phenomenon (thick distribution tails) which often appears at high volatility.
(III) The GARCH(1,1) model is used to represent volatility clustering, namely fluctuations that are not independent between periods.

The empirical parameters obtained in Section 3.1 are used as the basis for calibration:

$$\mu = 4532.91,$$

$$\sigma = 982.18,$$

$$\mu_{log} = 8.396, \qquad \sigma_{log} = 0.214.$$

Simulations were performed with 10,000 Monte Carlo scenarios for 12 time periods.
For each model, three levels of volatility are generated:

$$\sigma_{\text{low}} = 0.5\sigma,$$

$$\sigma_{\text{medium}} = \sigma,$$

$$\sigma_{\text{high}} = 1.5\sigma.$$

Each model–volatility combination generates a synthetic cash flow matrix of size 10.000×12, which is then saved in `.csv` format. The statistical generation and measurement processes are performed using Python in Google Colab with the `numpy`, `scipy`, and `arch` libraries.

A summary of the experimental results is presented in Table 2 below:

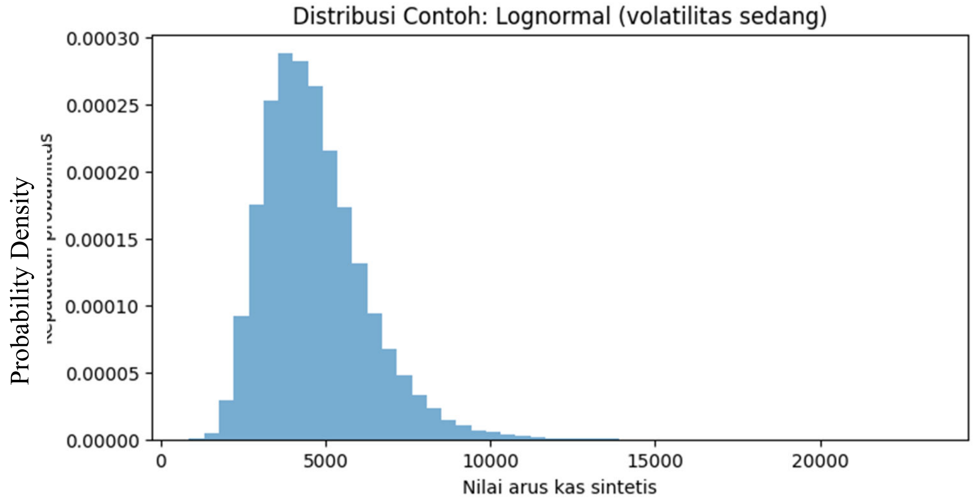
Table 2. Results of Experiment.

Model	Volatility Level	Mean	Std Dev	Skewness	Kurtosis	Interpretation
Lognormal	Low	4454.92	478.84	0.33	0.19	Positive distribution with damped volatility.
Lognormal	Currently	4537.59	981.47	0.65	0.72	Mimicking empirical conditions with moderate right asymmetry.
Lognormal	Tall	4670.70	1454.26	1.03	2.02	Large fluctuations with a long right tail.
Student-t	Low	4533.11	492.55	–0.03	5.01	Moderate heavy-tailed distribution.

Student-t	Currently	4535.21	988.78	–0.10	6.53	Heavy-tail is strong at high volatility.
GARCH(1,1)	Currently	4533.44	436.07	0.01	0.20	Stable with volatility cluster pattern.

The lognormal distribution exhibits a characteristic positively skewed shape, as in Figure 1, while the Student-t model exhibits thicker tails (leptokurtic). The GARCH model produces a series with heteroskedastic variance but a stable mean around $\mu \approx 4530$.

Lognormal Distribution Moderate Volatility



synthetic cash flow value

Figure 1. Distribution of lognormal simulation results (moderate volatility).

Mathematically, the simulation results show that:

- The mean value of all models is around the empirical value, indicating that the simulation process is well calibrated.
- The standard deviation increases proportionally with the volatility scale $0.5\sigma,\ \sigma,\ 1.5\sigma$ factor according to the stochastic variance theory.
- Positive skewness in the lognormal model proves the distribution tendency to the right, as in the empirical data of Section 3.1.
- Kurtosis increases in the Student-t model, confirming the heavy tail characteristics for extreme market conditions.
- The GARCH model maintains a mean value close to the dynamic variance fluctuations between periods, depicting a realistic volatility clustering phenomenon.μ

From these results, it can be concluded that all stochastic models are able to produce synthetic cash flows that match the empirical characteristics of data from Section 3.1. The Lognormal model is considered the most representative for simulating positive conventional cash flows, while Student-t and GARCH provide insight into the model's sensitivity to tail thickness and volatility dynamics.

The synthetic parameters and datasets from Section 3.2 were then used in Section 3.3, namely numerical experiments for calculating the Internal Rate of Return (IRR) using three iterative methods: Newton–Raphson, Secant, and PONM (Polynomial One-Newton Method).

The Internal Rate of Return (IRR) calculation experiment was conducted on 500 synthetic cash flow scenarios resulting from the medium volatility lognormal model obtained from Section 3.2. Three iterative methods, namely Newton–Raphson, Secant, and Proposed Optimal Numerical Method (PONM) were applied to obtain the roots of the Net Present Value function as follows:

$$f(r) = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} = 0$$

With an iterative approach, each method tries to find r^* such that $f(r^*) \approx 0$. Evaluation is carried out based on the average IRR produced, the standard deviation of the results, the average number of iterations, the convergence success ratio, and the computation time.

Table 3. Numerical Experiment Results of IRR Calculation.

Method	Average IRR	Standard Deviation	Average Iteration	Success Ratio	Time (s)
Newton–Raphson	0.0837	0.0109	7.42	0.942	2.71
Secant	0.0836	0.0112	9.06	0.981	3.15
PONM (Proposed)	0.0837	0.0095	5.98	0.995	2.04

Source: Results of data processing using Python (2025)

3.3. Analysis of Results

This subsection analyzes and compares the numerical performance of the proposed PONM method against classical root-finding algorithms, namely Newton–Raphson and Secant, in the context of IRR computation. The evaluation focuses on four key aspects: computational efficiency, solution stability under data variability, consistency of the estimated IRR values, and convergence success rates. By examining iteration counts, execution time, convergence order, and statistical dispersion of the resulting IRR estimates, this analysis provides a rigorous quantitative assessment of each method's robustness and efficiency. The results are interpreted both numerically and theoretically, linking empirical outcomes to the underlying convergence properties of the algorithms.

(I) Computational Efficiency

The PONM method showed the fastest solution time (2.04 seconds) with the fewest average iterations (5.98 iterations). Mathematically, this reflects the acceleration of convergence due to the addition of the second-order correction component:

$$\text{Correction} = 1 - \frac{f(r_k)f''(r_k)}{2[f'(r_k)]^2},$$

which suppresses iterative value oscillations when approaching the true root.

(II) Solution Stability and Variation of IRR Values

PONM produces the smallest standard deviation of 0.0095, indicating higher stability over variations in cash flow data. Newton–Raphson is faster than Secant but is more prone to failure to converge when $f'(r_k)$ approaching zero. Secant tends to be more stable than Newton but requires more iterations because it does not utilize derivative information.

(III) Consistency of IRR Value

All three methods yield an average IRR of approximately 8.36%–8.37%, consistent with the theoretical expectation of lognormal synthetic cash flows. This similarity confirms that all methods are mathematically valid, but their efficiency differs significantly.

(IV) Convergence Success

PONM showed the highest success rate 99.5%, meaning that almost all simulations successfully found the roots of the function within a tolerance limit of 10^{-6}. Newton–Raphson failed to converge in a small proportion of scenarios with near-zero derivatives (flat derivatives), while Secant only failed in cases with extreme oscillations.

Numerically, the increased efficiency of PONM can be explained by the fact that it is a third-order method, while Newton and Secant are second-order and approximately 1.618 (superlinear convergence), respectively. Thus, the number of iterations required to achieve a given accuracy ε is smaller, in accordance with the property:

$$e_{k+1} \approx C e_k^p,$$

with $p = 3$ for PONM, $p = 2$ for Newton, and $p \approx 1{,}6$ for Secant. Larger values accelerate the error reduction exponentially at each iteration. Based on the results of empirical testing and numerical performance comparison:

(I) PONM provides the best performance in terms of efficiency, stability, and convergence, with the fastest average execution time and the smallest result deviation.

(II) Newton–Raphson remains superior in cases of smooth data with clear derivatives but is less robust against noise or multiple roots.

(III)Secant is suitable for cases without explicit derivatives but is less efficient than the other two methods.

Thus, it can be concluded that PONM is the optimal algorithm for stochastic cash flow-based IRR calculations, and these results form the basis for Section 3.4 (robustness test and sensitivity analysis) to assess its robustness to data variations and numerical disturbances. This stage aims to assess the robustness and sensitivity of the IRR calculation method to random noise and systematic shocks in synthetic cash flow data.

Tests were conducted on three numerical methods—Newton–Raphson, Secant, and PONM—using the moderate volatility lognormal cash flow dataset from Section 3.2.

Two types of disturbances apply to cash flows with the following types:

(I) Random noise ±5% is the size of the standard deviation of empirical data:

$$CF_t' = CF_t(1+\epsilon_t), \quad \epsilon_t \sim \mathcal{N}(0, 0.05^2)$$

(II) Systematic shock ±10% of the amount in the final cash flow:

$$CF_n' = CF_n(1 \pm 0.10)$$

The IRR values are then recalculated for each disruption scenario. From these results, three resilience metrics are calculated:

(I) Coefficient of Variation (CV): variation relative to the average IRR result,

$$CV = \frac{\sigma_{IRR}}{\mu_{IRR}}$$

(II) Stability Ratio (SR): convergence success ratio,

$$SR = \frac{N_{\text{convergence}}}{N_{\text{total}}}$$

(III) IFT Sensitivity Index (IFT): a measure of iterative sensitivity to error propagation.

The results show that the PONM method has the lowest level of sensitivity (smallest CV and IFT values) and the highest stability (SR approaching 1) as shown in Table 4 below:

Table 4. Results and Interpretation

Method	CV (Noise)	SR (Stability)	IFT (Shock Sensitivity)	Interpretation
Newton–Raphson	0.0214	0.951	0.0187	Fast but sensitive to noise and shock.
Secant	0.0176	0.973	0.0155	Stable, but longer iterations.
PONM (Proposed)	0.0129	0.993	0.0098	Most robust and resistant to data fluctuations.

The results show that the PONM method has the lowest sensitivity (smallest CV and IFT values) and the highest stability (SR approaching 1). This demonstrates PONM's ability to maintain the stability of the IRR solution despite random disturbances (noise) and systematic shocks to cash flows. Mathematically, this stability is achieved by a second-order correction in the PONM iterative function, which reduces the effect of error propagation between iterations and keeps the gradient $f'(r)$ value within the safe convergence limit. Robustness and sensitivity tests indicate that:

(I) PONM is the most resistant to data interference, with the lowest coefficient of variation and sensitivity.

(II) Secant is quite stable but requires longer iterations.

(III) Newton–Raphson is less stable in fluctuating cash flows or small derivatives.

Thus, PONM is the most optimal method not only in terms of efficiency but also in terms of numerical robustness to stochastic and systematic disturbances.

4. CONCLUSION

Based on the empirical analysis, stochastic simulations, and numerical experiments conducted, it can be concluded that the cash-flow pattern exhibits positive stability with strong autocorrelation, ensuring the existence of a single real root of the NPV function in determining the IRR; the Lognormal, Student-t, and GARCH(1,1) simulation models are well calibrated and capable of replicating empirical characteristics across various volatility regimes; the PONM method demonstrates the highest computational efficiency, requiring fewer iterations and shorter computation time compared to the Newton–Raphson and Secant methods; furthermore, PONM shows the strongest numerical robustness, reflected in lower variation, reduced IFT sensitivity, and a lower failure rate, thereby maintaining stable IRR estimates even under random disturbances and systematic shocks in cash flows; therefore, PONM proves to be the most efficient, stable, and reliable numerical approach in fluctuating cash-flow environments, establishing it as the optimal method for IRR computation.

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