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Picture Fuzzy Subgroupoids and Ideals

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ABSTRACT

The object of this paper is to propound the notion of picture fuzzy subgroupoids and picture fuzzy ideals along with its example. Following the conception of picture fuzzy subgroupoids and picture fuzzy ideals as introduced, we have derived several appealing and interesting results from them.

Keywords: Picture fuzzy sets, Picture fuzzy subgroups, Picture fuzzy subgroupoids, Picture fuzzy ideals.

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1. INTRODUCTION

Nearly sixty years have elapsed since a researcher initiated a new notion in the realm of the mathematical world called fuzzy set (FS). After a gap of 21 years, researchers observed that when there is a membership value, then a non-membership should also exist, which leads to the surfacing of an intuitionistic fuzzy set (IFS). IFS deals with two kinds of membership functions referred to as membership and non-membership, whose sum always lies in [0,1]. IFS is treated as more general than the FS. Researchers were engrossed in further development of the theory by thinking that when there are membership and non-membership, then a neutral value should also be there, and this leads to the development of a picture fuzzy set (PFS). It was propounded by Cuong and Kreinovich [8]. It revolutionized the extensions of work done in fuzzy algebraic structures and intuitionistic fuzzy algebraic structures. The fuzzification of picture fuzzy algebraic structures was introduced by Dogra and Pal [10,11,12,13], and then the notion of picture fuzzy subring (PFSR), picture fuzzy subgroup (PFSG), picture fuzzy subspace, and picture fuzzy sub-hyperspace came into existence along with some interesting results.

In this paper, we wish to propound the conception of picture fuzzy subgroupoids and picture fuzzy ideals. Additionally, on these newly introduced notions, we have derived some interesting and appealing results over all of them.

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2. PRELIMINARIES

Definition 2.1. ([24]) For any universe X, a FS P over X is defined as $P = \{(b, \tau_P(b): b \in X\}$, where $\tau_P: X \to [0,1]$. The value $\tau_P(b)$ represent the membership degree of the element $b \in X$.

Definition 2.2. ([1]) For any universe X, a IFS P over X is defined as $P = \{(b, \tau_P(b), \vartheta_P(b): b \in X\}$, where $\tau_P(b) \in [0,1]$ represent the membership degree of the element $b \in X$, $\vartheta_P(b) \in [0,1]$ represent the non-membership degree of the element $b \in X$ with the condition $0 \leq \tau_P(b) + \vartheta_P(b) \leq 1$ for all $b \in X$.

Here, $H_p(b) = 1 - \left(\tau_P(b) + \vartheta_P(b)\right)$ is called the degree of hesitancy of $b \in X$, which excludes the membership degree and non-membership degree.

Definition 2.3. ([8]) For any set of universe X, a PFS P over X is defined as $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in X\}$, where $\tau_P(b) \in [0,1]$ represents the degree of positive membership of the element $b \in X$, $\vartheta_P(b) \in [0,1]$ represents the degree of neutral membership of the element $b \in X$ and $\eta_P(b)$ represent the degree of negative membership of the element $b \in X$ with the condition $0 \leq \tau_P(b) + \vartheta_P(b) + \eta_P(b) \leq 1$ for all $b \in X$.

Here, $H_p(b) = 1 - \left(\tau_P(b) + \vartheta_P(b) + \eta_P(b)\right)$ is called the degree of refusal membership of $b \in X$.

Definition 2.4. ([8]) Let P_1 and P_2 be two PFSs of X. Then the union of P_1 and P_2 is defined as $P_1 \cup P_2 = \{(b, \tau_{P_1 \cup P_2}(b), \vartheta_{P_1 \cup P_2}(b), \eta_{P_1 \cup P_2}(b): b \in X\}$, where $\tau_{P_1 \cup P_2}(b) = \max\{\tau_{P_1}(b), \tau_{P_2}(b)\}$, $\vartheta_{P_1 \cup P_2}(b) = \min\{\vartheta_{P_1}(b), \vartheta_{P_2}(b)\}$ and $\eta_{P_1 \cup P_2}(b) = \min\{\eta_{P_1}(b), \eta_{P_2}(b)\}$.

Definition 2.5. ([8]) Let P_1 and P_2 be two PFSs of X. Then the intersection of P_1 and P_2 is defined as $P_1 \cap P_2 = \{(b, \tau_{P_1 \cap P_2}(b), \vartheta_{P_1 \cap P_2}(b), \eta_{P_1 \cap P_2}(b): b \in X\}$, where $\tau_{P_1 \cap P_2}(b) = \min\{\tau_{P_1}(b), \tau_{P_2}(b)\}$, $\vartheta_{P_1 \cap P_2}(b) = \min\{\vartheta_{P_1}(b), \vartheta_{P_2}(b)\}$ and $\eta_{P_1 \cap P_2}(b) = \max\{\eta_{P_1}(b), \eta_{P_2}(b)\}$.

Definition 2.6. ([22]) A FS P of the group G be referred to as a fuzzy subgroupoid if $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}, \forall\, b, n \ in\ G$.

Definition 2.7. ([22]) The FS P of a group G be referred to as a fuzzy subgroup (FSG) if

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}$, $\forall\, b, n$ in G

(II) $\tau_P(b^{-1}) \geq \tau_P(b)$, $\forall\, b$ in G

Definition 2.8. ([4]) Let $\mathrm{P} = \{(b, \tau_P(b), \vartheta_P(b): b \in G\}$ be an IFS of the group G, where τ_p and ϑ_p are membership and non-membership functions respectively. Then, we call P an intuitionistic fuzzy subgroup (IFSG) of G if

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}, \forall\, b, n \in G$

(II) $\tau_P(b^{-1}) \geq \tau_P(b), \forall\, b \in G$

(III) $\vartheta_P(bn) \leq \max\{\vartheta_P(b), \vartheta_P(n)\}$, $\forall\, b, n \in G$

(IV) $\vartheta_P(b^{-1}) \leq \vartheta_P(b)$, $\forall\, b \in G$

Definition 2.9. ([16]) Let $\mathrm{P} = \{(b, \tau_P(b), \vartheta_P(b): b \in G\}$ be an IFS of the groupoid G, where τ_p and ϑ_p are membership and non-membership functions respectively. Then, we call P an intuitionistic fuzzy subgroupoid of G if

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}, \forall\, b, n \in G$

(II) $\vartheta_P(bn) \leq \max\{\vartheta_P(b), \vartheta_P(n)\}$, $\forall\, b, n \in G$

Definition 2.10. ([11]) Let C be a group and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ be a PFS in C. Then, the PFS P be referred to as the PFSG of C if the following axioms are satisfied:

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}$, $\vartheta_P(bn) \geq \min\{\vartheta_P(b), \vartheta_P(n)\}$, $\eta_P(bn) \leq \max\{\eta_P(b), \eta_P(n)\}$, $\forall\, b, n \in C$

(II) $\tau_Q(b^{-1}) \geq \tau_Q(b)$, $\vartheta_Q(b^{-1}) \geq \vartheta_Q(b)$, $\eta_Q(b^{-1}) \leq \eta_Q(b)$, $\forall\, b \in C$

Definition 2.11. ([10]) Let $f: C_1 \to C_2$ be a mapping and $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ be a PFS in C_2. Then the inverse image of Q under the map f is the PFS $f^{-1}(Q) = \left(\tau_{f^{-1}(Q)}, \vartheta_{f^{-1}(Q)}, \eta_{f^{-1}(Q)}\right)$, where $\tau_{f^{-1}(Q)}(b) = \tau_Q\left(f(b)\right)$, $\vartheta_{f^{-1}(Q)}(b) = \vartheta_Q\left(f(b)\right)$ and $\eta_{f^{-1}(Q)}(b) = \eta_Q\left(f(b)\right)$ for all $b \in C_1$.

Definition 2.12. ([10]) Let $f: C_1 \to C_2$ be an onto homomorphism and $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ be a PFS in C_1 . Then, the image of Q under the map f is the PFS $f(Q) = \left(\tau_{f(Q)}, \vartheta_{f(Q)}, \eta_{f(Q)}\right)$, where $\tau_{f(Q)}(b) = \sup_{a \in f^{-1}(b)} \tau_Q(a)$, $\vartheta_{f(Q)}(b) = \inf_{a \in f^{-1}(b)} \vartheta_Q(a)$ and $\eta_{f(Q)}(b) = \inf_{a \in f^{-1}(b)} \eta_Q(a)\ \forall\ b \in C_2$.

In the coming section, we will introduce the concept of picture fuzzy subgroupoids and picture fuzzy ideals in the domain of PFS.

3. PICTURE FUZZY SUBGROUPOIDS AND IDEAL

Definition 3.1. Let C be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ be a PFS in C. Then, the PFS P be referred to as the picture fuzzy subgroupoids of C if the following axioms are satisfied:

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}, \forall\ b, n \in C$

(II) $\vartheta_P(bn) \geq \min\{\vartheta_P(b), \vartheta_P(n)\}, \forall\ b, n \in C$

(III) $\eta_P(bn) \leq \max\{\eta_P(b), \eta_P(n)\}, \forall\ b, n \in C$

Example 3.1. Let $C = (\mathbb{R}, \cdot)$ be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ be a PFS in C characterised by

$$\tau_P(x) = \begin{cases} 0.8, & x = 0 \\ 0.7, & \text{otherwise} \end{cases}$$
$$\vartheta_P(x) = \begin{cases} 0.9, & x = 0 \\ 0.8, & \text{otherwise} \end{cases}$$
$$\eta_P(x) = \begin{cases} 0.2, & x = 0 \\ 0.3, & \text{otherwise} \end{cases}$$

Here, $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}$, $\vartheta_P(bn) \geq \min\{\vartheta_P(b), \vartheta_P(n)\}$ and $\eta_P(bn) \leq \max\{\eta_P(b), \eta_P(n)\}, \forall\ b, n \in C$. Thus, P is the picture fuzzy subgroupoids of C.

Definition 3.2. Let C be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ be a PFS in C. Then, the PFS P be referred to as

(I) picture fuzzy left ideal (PFLI) of C if $\tau_P(bn) \geq \tau_P(n)$, $\vartheta_P(bn) \geq \vartheta_P(n)$ and $\eta_P(bn) \leq \eta_P(n), \forall\ b, n \in C$.

(II) picture fuzzy right ideal (PFRI) of C if $\tau_P(bn) \geq \tau_P(b)$, $\vartheta_P(bn) \geq \vartheta_P(b)$ and $\eta_P(bn) \leq \eta_P(b),\ \forall\ b, n \in C$.

(III) picture fuzzy ideal (PFI) of C if it is both PFLI and PFI.

It is clear that P is an PFI of C iff $\tau_P(bn) \geq \max\{\tau_P(b), \tau_P(n)\}$, $\vartheta_P(bn) \geq \max\{\vartheta_P(b), \vartheta_P(n)\}$

and $\eta_P(bn) \leq \min\{\eta_P(b), \eta_P(n)\}$, $\forall\ b, n \in C$. Moreover, PFI (left or right) is picture fuzzy subgroupoids of C. Note that for any picture fuzzy subgroupoid P of C we have $\tau_P(b^n) \geq \tau_P(n)$, $\vartheta_P(b^n) \geq \vartheta_P(n)$ and $\eta_P(b^n) \leq \eta_P(n)$ $\forall\ b \in C$, where b^n is any composite of b's.

Remark 3.1.

(I) If a PFS $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ is a picture fuzzy subgroupoid of C, then $\tau_P(b)$, $\vartheta_P(b)$ and ${\eta^c}_P(b)$ are fuzzy subgroupoids of C.

(II) If a PFS $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$ is a PFI (PFLI or PFRI) of C, then $\tau_P(b)$, $\vartheta_P(b)$ and ${\eta^c}_P(b)$ are fuzzy (left or right) ideal of C.

Theorem 3.1. Let C be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$, $Q = \{(b, \tau_Q(b), \vartheta_Q(b), \eta_Q(b): b \in C\}$ be two picture fuzzy subgroupoids of C. Then $P \cap Q$ is a picture fuzzy subgroupoid of C.

Proof: Let $P \cap Q = W = \{(b, \tau_W(b), \vartheta_W(b), \eta_W(b): b \in C\}$, then $\tau_W(b) = \min\{\tau_P(b), \tau_Q(b)\}$, $\vartheta_W(b) = \min\{\vartheta_P(b), \vartheta_Q(b)\}$ and $\eta_W(b) = \max\{\eta_P(b), \eta_Q(b)\}$ $\forall\ b \in C$. Since P and Q are picture fuzzy subgroupoids of C, therefore

(I) $\tau_P(bn) \geq \min\{\tau_P(b), \tau_P(n)\}$ and $\tau_Q(bn) \geq \min\{\tau_Q(b), \tau_Q(n)\}$, $\forall\ b, n \in C$

(II) $\vartheta_P(bn) \geq \min\{\vartheta_P(b), \vartheta_P(n)\}$ and $\vartheta_Q(bn) \geq \min\{\vartheta_Q(b), \vartheta_Q(n)\}$, $\forall\ b, n \in C$

(III) $\eta_P(bn) \leq \max\{\eta_P(b), \eta_P(n)\}$ and $\eta_Q(bn) \leq \max\{\eta_Q(b), \eta_Q(n)\}$, $\forall\ b, n \in C$

Now $\forall\ b, n \in C$, we have

$$\begin{aligned}
\tau_W(bn) &= \min\{\tau_P(bn), \tau_Q(bn)\} \\
&\geq \min\{\min\{\tau_P(b), \tau_P(n)\}, \min\{\tau_Q(b), \tau_Q(n)\}\} \\
&= \min\{\min\{\tau_P(b), \tau_Q(b)\}, \min\{\tau_P(n), \tau_Q(n)\}\} \\
&= \min\{\tau_W(b), \tau_W(n)\},
\end{aligned}$$

$$\begin{aligned}
\vartheta_W(bn) &= \min\{\vartheta_P(bn), \vartheta_Q(bn)\} \\
&\geq \min\{\min\{\vartheta_P(b), \vartheta_P(n)\}, \min\{\vartheta_Q(b), \vartheta_Q(n)\}\} \\
&= \min\{\min\{\vartheta_P(b), \vartheta_Q(b)\}, \min\{\vartheta_P(n), \vartheta_Q(n)\}\} \\
&= \min\{\vartheta_W(b), \vartheta_W(n)\},
\end{aligned}$$

and

$$\eta_W(bn) = \max\{\eta_P(bn), \eta_Q(bn)\}$$

$$\leq \max\{\max\{\eta_P(b), \eta_P(n)\}, \max\{\eta_Q(b), \eta_Q(n)\}\}$$

$$= \max\{\max\{\eta_P(b), \eta_Q(b)\}, \max\{\eta_P(n), \eta_Q(n)\}\}$$

$$= \max\{\eta_W(b), \eta_W(n)\}$$

Hence, $W = P \cap Q$ is a picture fuzzy subgroupoid of C.

Theorem 3.2. Let C be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$, $Q = \{(b, \tau_Q(b), \vartheta_Q(b), \eta_Q(b): b \in C\}$ be two picture fuzzy (left or right) ideals of C. Then $P \cap Q$ is a picture fuzzy (left or right) ideal of C.

Proof: Let $P \cap Q = W = \{(b, \tau_W(b), \vartheta_W(b), \eta_W(b): b \in C\}$, then $\tau_W(b) = \min\{\tau_P(b), \tau_Q(b)\}$, $\vartheta_W(b) = \min\{\vartheta_P(b), \vartheta_Q(b)\}$ and $\eta_W(b) = \max\{\eta_P(b), \eta_Q(b)\}$ $\forall\, b \in C$. Let P and Q are PFLI of C, then $\forall\, b, n \in C$

$$\tau_W(bn) = \min\{\tau_P(bn), \tau_Q(bn)\}$$

$$\geq \min\{\tau_P(n), \tau_Q(n)\}$$

$$= \tau_W(n),$$

$$\vartheta_W(bn) = \min\{\vartheta_P(bn), \vartheta_Q(bn)\}$$

$$\geq \min\{\vartheta_P(n), \vartheta_Q(n)\}$$

$$= \vartheta_W(n)$$

and

$$\eta_W(bn) = \max\{\eta_P(bn), \eta_Q(bn)\}$$

$$\leq \max\{\eta_P(n), \eta_Q(n)\}$$

$$= \eta_W(n)$$

Hence, $W = P \cap Q$ is a PFLI of C. By following the same footsteps, we can show that the intersection of two PFRI is a PFRI.

Theorem 3.3. Let C be a groupoid and $P = \{(b, \tau_P(b), \vartheta_P(b), \eta_P(b): b \in C\}$, $Q = \{(b, \tau_Q(b), \vartheta_Q(b), \eta_Q(b): b \in C\}$ be two picture fuzzy (left or right) ideals of C. Then $P \cup Q$ is a picture fuzzy (left or right) ideal of C.

Proof: Let $P \cup Q = W = \{(b, \tau_W(b), \vartheta_W(b), \eta_W(b): b \in C\}$, then $\tau_W(b) = \max\{\tau_P(b), \tau_Q(b)\}$, $\vartheta_W(b) = \min\{\vartheta_P(b), \vartheta_Q(b)\}$ and $\eta_W(b) = \min\{\eta_P(b), \eta_Q(b)\}$ $\forall\ b \in C$. Let P and Q are PFLI of C, then $\forall\ b, n \in C$

$$\tau_W(bn) = \max\{\tau_P(bn), \tau_Q(bn)\}$$

$$\geq \max\{\tau_P(n), \tau_Q(n)\}$$

$$= \tau_W(n),$$

$$\vartheta_W(bn) = \min\{\vartheta_P(bn), \vartheta_Q(bn)\}$$

$$\geq \min\{\vartheta_P(n), \vartheta_Q(n)\}$$

$$= \vartheta_W(n)$$

and

$$\eta_W(bn) = \min\{\eta_P(bn), \eta_Q(bn)\}$$

$$\leq \min\{\eta_P(n), \eta_Q(n)\}$$

$$= \eta_W(n)$$

Hence, $W = P \cup Q$ is a PFLI of C. By following the same footsteps, we can show that the union of two PFRI is a PFRI.

Theorem 3.3. Let C_1 and C_2 be two groupoids. Let $f: C_1 \to C_2$ be an onto homomorphism. Then we have that

(I) if $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ is a picture fuzzy subgroupoid of C_2, then $f^{-1}(Q)$ is a picture fuzzy subgroupoid of C_1

(II) if $P = (\tau_P, \vartheta_P, \eta_P)$ is a picture fuzzy subgroupoid of C_1, then $f(P)$ is a picture fuzzy subgroupoid of C_2.

Proof: (I) Let $f^{-1}(Q) = \left(\tau_{f^{-1}(Q)}, \vartheta_{f^{-1}(Q)}, \eta_{f^{-1}(Q)}\right)$, where $\tau_{f^{-1}(Q)}(b) = \tau_Q\left(f(b)\right)$, $\vartheta_{f^{-1}(Q)}(b) = \vartheta_Q\left(f(b)\right)$ and $\eta_{f^{-1}(Q)}(b) = \eta_Q\left(f(b)\right)$, $\forall\ b \in C_1$. Then $\forall\ b, n \in C_1$, we have

(I) $\tau_{f^{-1}(Q)}(bn) = \tau_Q\left(f(bn)\right)$

$$= \tau_Q(f(b)f(n))$$

$$\geq \min\{\tau_Q(f(b)), \tau_Q(f(n))\}$$

$$= \min\{\tau_{f^{-1}(Q)}(b), \tau_{f^{-1}(Q)}(n)\}$$

i.e., $\tau_{f^{-1}(Q)}(bn) \geq \min\{\tau_{f^{-1}(Q)}(b), \tau_{f^{-1}(Q)}(n)\}$

Similarly, $\vartheta_{f^{-1}(Q)}(bn) \geq \min\{\vartheta_{f^{-1}(Q)}(b), \vartheta_{f^{-1}(Q)}(n)\}$ and $\eta_{f^{-1}(Q)}(bn) \leq \max\{\eta_{f^{-1}(Q)}(b), \eta_{f^{-1}(Q)}(n)\}$

Therefore, $f^{-1}(Q)$ is a picture fuzzy subgroupoid of C_1.

(II) Similar to proof of part (I).

Theorem 3.4. Let C_1 and C_2 be two groupoids. Let $f: C_1 \to C_2$ be an onto homomorphism. Then we have that

(I) if $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ is a PFLI of C_2, then $f^{-1}(Q)$ is a PFLI of C_1

(II) if $P = (\tau_P, \vartheta_P, \eta_P)$ is a PFLI of C_1, then $f(P)$ is an PFLI of C_2.

(III) if $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ is a PFRI of C_2, then $f^{-1}(Q)$ is a PFRI of C_1

(IV) if $P = (\tau_P, \vartheta_P, \eta_P)$ is a PFRI of C_1, then $f(P)$ is an PFRI of C_2.

Proof: Let $f^{-1}(Q) = \left(\tau_{f^{-1}(Q)}, \vartheta_{f^{-1}(Q)}, \eta_{f^{-1}(Q)}\right)$, where $\tau_{f^{-1}(Q)}(b) = \tau_Q\big(f(b)\big)$, $\vartheta_{f^{-1}(Q)}(b) = \vartheta_Q\big(f(b)\big)$ and $\eta_{f^{-1}(Q)}(b) = \eta_Q\big(f(b)\big)$, $\forall\, b \in C_1$.

(I) Let $Q = (\tau_Q, \vartheta_Q, \eta_Q)$ is a PFLI of C_2, then $\forall\, b, n \in C_1$

$$\begin{aligned}
\tau_{f^{-1}(Q)}(bn) &= \tau_Q\big(f(bn)\big) \\
&= \tau_Q(f(b)f(n)) \\
&\geq \tau_Q(f(n)) \\
&= \tau_{f^{-1}(Q)}(n)
\end{aligned}$$

i.e., $\tau_{f^{-1}(Q)}(bn) \geq \tau_{f^{-1}(Q)}(n)$

Similarly, $\vartheta_{f^{-1}(Q)}(bn) \geq \vartheta_{f^{-1}(Q)}(n)$ and $\eta_{f^{-1}(Q)}(bn) \leq \eta_{f^{-1}(Q)}(n)$

Therefore, $f^{-1}(Q)$ is a PFLI of C_1.

By following the same footsteps, we can show that the remaining parts of the results also hold good.

4. CONCLUSION

Following the notions introduced in this paper, we can extend the results in several directions under the umbrella picture fuzzy set and picture fuzzy groupoid to erect a bigger structure.

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