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The SIR Model Describes the Spread of Leptospirosis in Pati, Indonesia.

Ade Ima Afifa Himayati[1], Mursidah[2], Nur Hakimah[3] , Ivanna Isty Nursani[4]

[1,2,3,4] Muhammadiyah University of Kudus, Indonesia

E-mail address: 42022130012@std.umku.ac.id

ABSTRACT

Leptospirosis is a bacterial disease that spreads through the urine of infected animals. Humans can get leptospirosis through direct contact with the urine of infected animals or through water, soil, and food contaminated with animal urine (zoonosis). This condition is most common in warm climates, like Indonesia. Over the last 5 years, 2023 is the year with the highest number of Leptospirosis cases in Central Java, namely 884 cases, with 37 cases of Leptospirosis occurring in Pati. The number of fatalities in Central Java due to this disease was 139, of which 18 were in Pati. This means that by the end of 2023, there will be 19 victims who will recover from this disease, even though the CFR (case fatality rate) of this disease in Pati reaches 48.65%. In Pati, it was recorded that from 2019 to 2023 there was always an increase in cases of leptospirosis: in 2019 there were 5 cases, in 2020 there were 24 cases, in 2021 there were 16 cases, in 2022 there were 28 cases, and in 2023 there were 37 cases. The research method used is the SIR model, based on the results of equilibrium point calculations and basic reproduction to make final conclusions. The data was taken from the number of residents infected with Leptospirosis reported to the Central Java Health Service and population data from the Central Statistics Agency of Pati Regency, Central Java, Indonesia. The calculation results show the stability of the equilibrium point, the instability of the endemic point, and the basic reproduction value $= 0{,}00001 < 1$), so it can be concluded that one day in the future,
the leptospirosis disease in Pati will disappear by itself.

Keywords: Leptospirosis, Zoonoses, SIR model, CFR (case fatality rate),Endemic, Equilibrium Point, Basic Reproduction.

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1. INTRODUCTION

According to Dr. Pittara (2023), Leptospirosis is a disease caused by Leptospira bacteria. This bacteria can spread through the urine or blood of infected animals. Symptoms of leptospirosis are similar to flu symptoms but are more severe and accompanied by swelling in the feet and hands and yellow skin. If not treated properly, leptospirosis can cause damage to internal organs, even life-threatening damage.

Leptospirosis is caused by the bacteria Leptospira interrogans, which lives for several years in the kidneys. Some animals that can spread Leptospira bacteria are dogs, pigs, horses, cows, and mice. Leptospira bacteria can, at any time, come out with urine, thereby contaminating water and soil. In water and soil, these bacteria can survive for several months or years. Transmission of Leptospira bacteria to humans can occur due to direct contact between the skin and the urine of animals carrying the bacteria. Contact between the skin and water and soil contaminated with the urine of animals carrying the bacteria, Consuming food or drink contaminated with the urine of animals carrying the Leptospira bacteria can enter the body through open wounds, whether small wounds such as abrasions or large wounds such as lacerations. This bacteria can also enter through the eyes, nose, mouth, and digestive tract. Leptospirosis can also be transmitted between humans through breast milk or sexual intercourse, but this case is very rare.

Leptospirosis is a disease that is of little concern but has quite dangerous impacts compared to other infectious diseases. Data in Central Java for 2024 reports from districts and cities: as of the end of March, there were 124 cases with 23 people dying. What is possible will be even more than in 2023, when the incidence of leptospirosis cases is 2.37 per 1000 population, so there were 884 cases in one year and 139 people died. (Kominfo., 2024)

Among the 884 leptospirosis cases in Central Java, 37 of them were residents of Pati District. The Pati District Health Service (Dinkes) stated that from year to year, cases of leptospirosis always increase. Head of the Pati Health Office, Aviani Tritanti, said that this high number of cases should be a serious concern because leptospirosis can cause death in humans. (Agil., 2023)

Leptospirosis disease analysis uses modified SIR modeling according to the assumptions made for the disease under study. SIR modeling has been widely used to analyze the spread of other diseases. As done by a researcher named Zach Yarus in his journal entitled "A Mathematical Look at the Ebola Virus." (Yarus., 2012). Apart from that, in 2015, Benny Yong and Livia Owen researched the Mers-Cov disease in their journal entitled "Model of the Spread of the Infectious Disease MERS-CoV: An Anticipatory Step for Prospective Indonesian Umrah/Hajj Pilgrims." In his journal, he discussed the spread of the Mers-Cov disease that occurred among Hajj pilgrims from Indonesia who were performing their pilgrimage in Saudi Arabia.

In the journal "Effectiveness of Large-Scale Social Restrictions (PSBB) in Bekasi City in Overcoming COVID-19 Using the Susceptible-Infected-Recovered (SIR) Model," the effectiveness of PSBB in Bekasi City is explained. This research tries to use the Susceptible-Infected-Recovered (SIR) model to measure the rate of spread of COVID-19. The results show that the rate of decline in cases of infection with beta and gamma is 0.071 and 0.05, respectively, and is predicted to end in June 2020. (Handayanto, R. T., & Herlawati, H., 2020).

In the special region of Yogyakarta, research was also carried out on COVID-19 in the journal "SIR-based Models in Early Prediction of the Spread of COVID-19 in the Special Region of Yogyakarta (DIY)." The modeling was carried out based on the SIR model, whose parameters were estimated based on the data. Using this model, two scenarios will be studied, which are optimistic and pessimistic. (Adi-Kusumo, F., Susyanto, N., Endrayanto, I., & Meliala, A., 2020).

The SIR model is also used in the journal "Dynamics of HIV/AIDS Development in North Sulawesi." Using the SIR (Susceptible-Infected-Recovered) Nonlinear Differential Equation Model," which contains research to determine the dynamics of HIV/AIDS development in North Sulawesi using the SIR nonlinear differential equation model. The results of the stability analysis based on the eigenvalues of the Jacobi matrix obtained a semi-stable disease-free fixed point and also obtained a basic reproduction number for HIV/AIDS in North Sulawesi of 4.155. These results indicate that there will be an HIV/AIDS epidemic in the next 100 years. (Tjolleng, A., Komalig, H. A., & Prang, J. D., 2013).

Based on the description above, the aim of this research is to analyze how dangerous this disease will be in the future according to the number of Pati Regency residents who are susceptible, namely individuals who have the potential to be infected with Leptospirosis. Conclusions were drawn using SIR modeling to precisely calculate the equilibrium point and basic reproduction of Leptospirosis in Pati in 2021–2023.

2. MATERIALS AND METHODS

The research was conducted using qualitative methods. Leptospirosis case data was taken from the Central Java Health Service, and population data from Pati Regency was taken from the Pati Regency Central Statistics Agency. In mathematical modeling, the mathematical model that will be used as the basic model and reference in this research is determined, namely the SIR (Susceptible, Infected, and Recovered) model. In this research, the following assumptions were used:

- Individuals who recover have immunity to the disease, or it can be said that individuals who recover cannot become susceptible again.
- The disease is not fatal.
- The incubation period is very short.

From the above assumptions, the following transfer diagram is obtained:

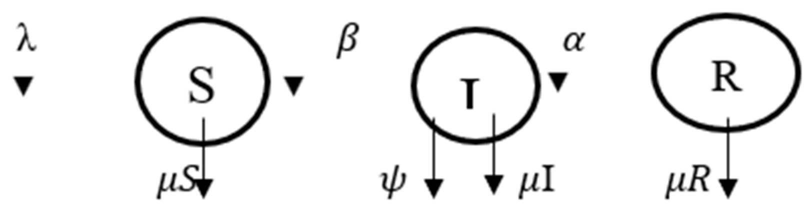


Figure 1. Transition diagram of the spread of leptospirosis.

From this diagram, it can be written into the following equations:

$$\frac{dS}{dt} = \lambda - \beta SI - \mu S \qquad (1)$$

$$\frac{dI}{dt} = \beta SI - \mu I - \alpha I - \psi I \qquad (2)$$

$$\frac{dR}{dt} = \alpha I - \mu R \qquad (3)$$

Description of each variable:

S = Number of susceptible individuals
I = Number of infected individuals
R = Number of individuals recovered
λ = Birth or migration rate
β = Transmission rate
α = Recovery rate
μ = Natural death rate
ψ = Death rate due to infection

From the equations above, it can be found:

2.1. Disease-Free Equilibrium Point

Assume the number of infected individuals I = 0, then from equations (1) and (3) it is made equal to 0 to obtain the equilibrium points S and R.

Equation (1)

$$\frac{dS}{dt} = \lambda - \beta SI - \mu S$$
$$0 = \lambda - \beta SI - \mu S$$
$$0 = \lambda - \beta S(0) - \mu S$$
$$0 = \lambda - \mu S$$
$$\lambda = -\mu S$$
$$S = \frac{\lambda}{\mu}$$

Equation (3)

$$\frac{dR}{dt} = \alpha I - \mu R$$
$$0 = \alpha I - \mu R$$
$$0 = \alpha(0) - \mu R$$
$$0 = -\mu R$$
$$\frac{0}{-\mu} = R$$
$$0 = R$$

From the substitution results above, the disease-free equilibrium point is obtained

$$TE_0 = (S, I, R) = \left(\frac{\lambda}{\mu}, 0{,}0\right)$$

2.2. Endemic Equilibrium Point

We can get the endemic equilibrium point from

$$\beta S - \mu - \alpha - \psi = 0$$

$$\beta S = \mu + \alpha + \psi$$

$$S = \frac{\mu + \alpha + \psi}{\beta}$$

It can be concluded that point $S^* = \frac{\mu+\alpha+\psi}{\beta}$, then to find point I^* substitute S^* into equation (1), which is equal to zero.

$$\frac{dS}{dt} = \lambda - \beta SI - \mu S$$

$$0 = \lambda - \beta\left(\frac{\mu+\alpha+\psi}{\beta}\right) I - \mu\left(\frac{\mu+\alpha+\psi}{\beta}\right)$$

$$I(\mu + \alpha + \psi) = \lambda - \mu\left(\frac{\mu+\alpha+\psi}{\beta}\right)$$

$$I = \frac{\lambda}{(\mu+\alpha+\psi)} - \frac{\mu}{\beta}$$

Obtain $I^* = \frac{\lambda}{(\mu+\alpha+\psi)} - \frac{\mu}{\beta}$, then find R^* by substituting I^* in equation (3), which is equal to zero.

$$\frac{dR}{dt} = \alpha I - \mu R$$

$$0 = \alpha\left(\frac{\lambda}{(\mu+\alpha+\psi)} - \frac{\mu}{\beta}\right) - \mu R$$

$$\mu R = \frac{\alpha\lambda}{(\mu+\alpha+\psi)} - \frac{\alpha\mu}{\beta}$$

$$R = \frac{\alpha\lambda}{\mu(\mu+\alpha+\psi)} - \frac{\alpha}{\beta}$$

After obtaining R*, the endemic equilibrium point is known, $(S^*, I^*, R^*) = \left(\frac{\mu+\alpha+\psi}{\beta}, \frac{\lambda}{(\mu+\alpha+\psi)} - \frac{\mu}{\beta}, \frac{\alpha\lambda}{\mu(\mu+\alpha+\psi)} - \frac{\alpha}{\beta}\right)$.

2.3. Basic Reproduction Number (R_0)

In this article, $R_0$is searched using the New Generation Matrices (NGM) method. The steps to determine the basic reproduction number (R_0) using New Generation Matrices (NGM) are as follows:

- Linearize the infected subsystem around the disease-free equilibrium point. This linear system is represented by the Jacobi matrix (J).

$$J(I) = \frac{\partial}{\partial I}(\beta SI - \mu I - \alpha I - \psi I)$$
$$= \beta S - \mu - \alpha - \psi.$$
$$J\left(\frac{\lambda}{\mu}, 0,0\right) = \beta\left(\frac{\lambda}{\mu}\right) - \mu - \alpha - \psi.$$

- Decompose the Jacobi matrix *(J)* into a transmission matrix *(F)* and a transition matrix *(V)*. The Transmission Matrix is a matrix with entries that describe the emergence of new infections, while the Transition Matrix is a matrix with entries that describe changes in the infected population.

$$J = F - V$$
$$= \beta\left(\frac{\lambda}{\mu}\right) - (\mu + \alpha + \psi).$$

So it is obtained $F = \beta\left(\frac{\lambda}{\mu}\right)$ and $V = \mu + \alpha + \psi$.

- Count R_0 with $R_0 = \rho\left(FV^{-1}\right)$ is the spectral radius or largest eigenvalue of the matrix $R_0 = \rho\left(FV^{-1}\right)$.

$$R_0 = \rho\left(FV^{-1}\right)$$
$$= \rho\left(\beta\left(\frac{\lambda}{\mu}\right)\frac{1}{\mu+\alpha+\psi}\right)$$
$$= \rho\left(\frac{\beta\lambda}{\mu(\mu+\alpha+\psi)}\right)$$
$$= \frac{\beta\lambda}{\mu(\mu+\alpha+\psi)}$$

With$R_0 = \frac{\beta\lambda}{\mu(\mu+\alpha+\psi)}$ There are the following provisions:

- If $R_0 < 1$, then the disease-free equilibrium point is locally asymptotically stable.
- If $R_0 > 1$, then the disease-free equilibrium point is unstable, and the equilibrium point is endemic(S^*, I^*, R^*) locally asymptotically stable.

3. RESULT AND DISCUSSION

Data on leptospirosis cases that occurred in Central Java in the last 5 years.

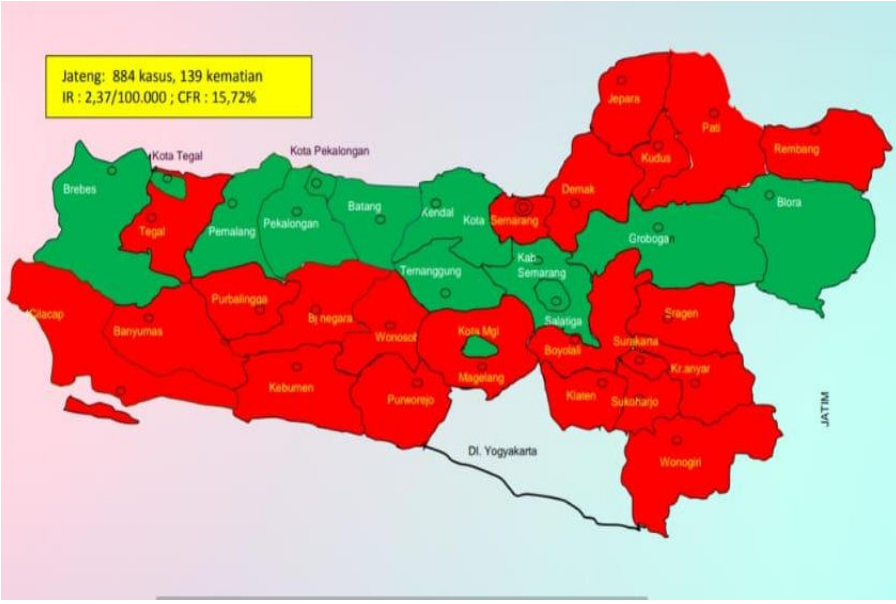
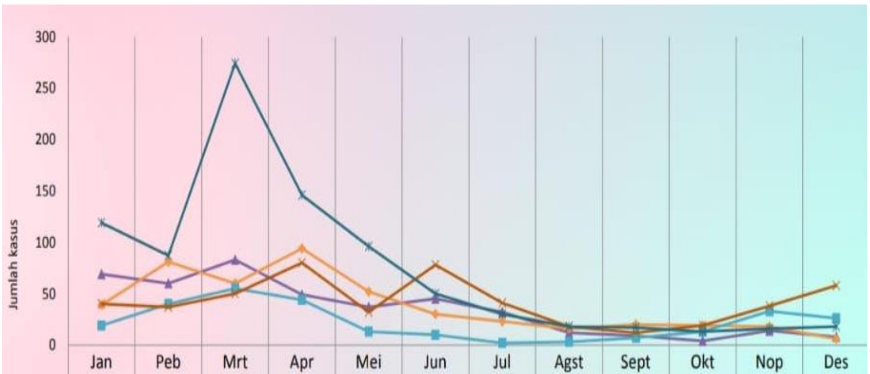


Figure 2. Map of the Distribution of Leptospirosis, Central Java Province, TH. 2023

There are more than 20 districts in Central Java that are affected by leptospirosis cases, from the easternmost region, namely Cilacap Regency, to the westernmost region of Central Java, namely Rembang, and only the northern part of Central Java is not affected by this disease.



	Jan	Peb	Mrt	Apr	Mei	Jun	Jul	Agst	Sept	Okt	Nop	Des
2020	69	60	83	49	37	45	32	12	9	4	14	8
2019	39	81	60	94	52	30	23	16	20	19	18	6
2021	19	40	55	44	13	10	2	3	7	13	33	26
2022	40	37	50	80	32	78	41	18	12	19	38	58
2023	119	87	274	146	96	50	30	18	17	13	16	18

Figure 3. Trend in the Number of Leptospirosis Cases Per Month, TH. 2019 to 2023, in Central Java.

From the picture above, you can see in detail the number of Leptospirosis cases every month from 2019 to 2023 in Central Java. In this picture, it can also be seen that cases of leptospirosis have increased very significantly in March 2023, reaching 274 cases, compared to the number of cases in February 2023, which only reached 87 cases.

Figure 4. Trends in Death Cases and Case Fatality Rate (CFR) of Leptospirosis, Central Java Province, 2019 to 2019. 2023

From the bar diagram image, it can be seen that the morbidity rate per 100,000 population of leptospirosis cases in Central Java decreased from 2019 to 2021. Until 2022, it rose again and increased in 2023, causing 139 people to die.

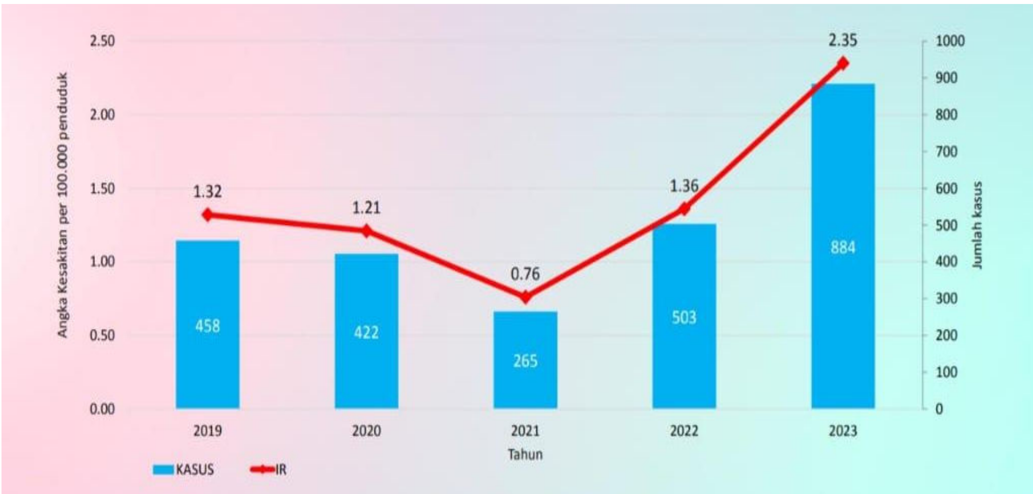


Figure 5. Trends in New Cases and Incidence Rate (IR) of Leptospirosis in Central Java Province from 2019 to 2023

From the data above, it can be seen how dangerous leptospirosis is if left alone. In this research, we will try to find a model for the spread of leptospirosis in Pati Regency to find out the possibilities that will occur.

3.1. Numerical Analysis

The analysis in this study was carried out using the SIR model; the data used came from the spread of leptospirosis in starch in 2021–2023. Using the document study method, data on cases of leptospirosis and population were obtained from the Central Java Health Service and the Pati Regency Central Statistics Agency.

Table 1. Table of BPS Pati Population 2021-2023

Year	Population birth (Soul)	Population death (Soul)	Total population (Soul)
2021	25.161	10.920	1.330.983
2022	28.997	55.699	1.339.572
2023	27.541	66.177	1.359.364

The table above is taken from population data from the Pati Regency Central Statistics Agency, which contains births, deaths, and population from 2021 to 2023. For the death data above, it is assumed that the cause of death is not only leptospirosis but also other causes.

Table 2. Table of distribution of leptospirosis 2021-2023

Year	Leptospyosis infection (Soul)	Death from Leptospirosis (Life)
2021	16	4
2022	28	7
2023	37	18

From Table 1, the proportion of natural births can be analyzed (λ) by dividing the number of births by the total population in Pati Regency and the death rate (μ) by dividing the number of deaths by the total population. To get the other three parameters, namelyβ, α, ψ, can be analyzed from both tables. Rate of new Leptospirosis infections (β) obtained by dividing the number of Leptospirosis infections by the population of Pati Regency. And finally, the recovery rate (α) or death (ψ) obtained by dividing the number of recoveries or deaths by the number of Leptospirosis infections.

Table 3. Table of Parameters for the Spread of Leptospirosis in Pati

Parameter	Mark	Unit
λ	0,02027	Soul
μ	0,03295	1/ people
β	0,00002	1/ people
α	0,64197	1/ people
ψ	0,35802	1/ people

With assumption $I = 0$, the disease-free equilibrium point can be obtained by $TE_0 = (S, I, R) = (\frac{\lambda}{\mu}, 0,0)$

$$S = \frac{\lambda}{\mu} = \frac{0,02027}{0,03295} = 0,61517$$

So the disease-free equilibrium point from the data above is $= (0,61517; 0; 0)$.

Then look for the endemic equilibrium point using the formula
$(S^*,I^*,R^*) = \left(\frac{\mu+\alpha+\psi}{\beta}, \frac{\lambda}{(\mu+\alpha+\psi)}\frac{\mu}{\beta}, \frac{\alpha\lambda}{\mu(\mu+\alpha+\psi)} - \frac{\alpha}{\beta}\right)$.

$$S^* = \frac{0,03295+0,64197+0,35802}{0,00002}$$
$$= \frac{1,03294}{0,00002}$$
$$= 51,64700$$

$$I^* = \frac{\lambda}{(\mu+\alpha+\psi)} - \frac{\mu}{\beta}$$
$$= \frac{0,02027}{0,03295+0,64197+0,35802} - \frac{0,03295}{0,00002} \qquad = \frac{0,02027}{1,03294} - \frac{0,03295}{0,00002}$$
$$= -1.647,48037$$

$$R^* = \frac{\alpha\lambda}{\mu(\mu+\alpha+\psi)} - \frac{\alpha}{\beta}$$
$$= \frac{0,64197(0,02027)}{0,03295(0,03295+0,64197+0,35802)} - \frac{0,64197}{0,00002}$$
$$= \frac{0,01301}{0,03295(1,03294)} - \frac{0,64197}{0,00002}$$
$$= -32.098,11767$$

So the endemic equilibrium point is
$(S^*,I^*,R^*) = (51,64700; -1.647,48037; -32.098,11767)$ Unstable

Next, find the basic reproduction number) by using the formula $\frac{\beta\lambda}{\mu(\mu+\alpha+\psi)}$.

$$R_0 = \frac{\beta\lambda}{\mu(\mu+\alpha+\psi)}$$

$$= \frac{0{,}00002(0{,}02027)}{0{,}03295(0{,}03295+0{,}64197+0{,}35802)} \qquad = \frac{0{,}0000004054}{0{,}03295(1{,}03294)}$$

$$= 0{,}00001.$$

From the results of calculating the basic reproduction number(R_0) above, it is known $R_0 = 0{,}00001 < 1$, the disease-free equilibrium point is locally asymptotically stable.

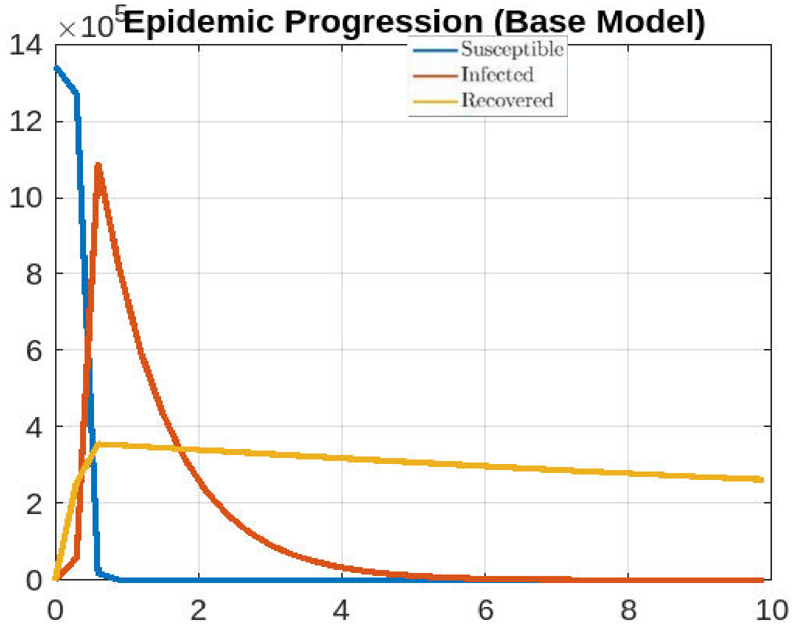


Figure 6. System Trajectory with $R_0 < 1$

Also proven using MathLAB software. With $R_0 = 0{,}00001 < 1$, It can be seen in Figure 6 above that susceptible, infected, and recovered cases of leptospirosis in Pati will continue to decrease until they approach 0. Thus, it can be concluded that by maintaining R_0 Maintaining this figure, leptospirosis in Pati will not spread to become endemic and will even disappear from Pati Regency.

4. CONCLUSIONS

Modeling of leptospirosis using the SIR (Susceptible, Infected, and Recovered) model was applied to the number of leptospirosis cases in Pati Regency and the total population in the district, taken from the Central Java Health Service and population data from the Central Statistics Agency of Pati Regency, Central Java, Indonesia, describe the condition of this population against Leptospirosis disease in the future.

With the results of the endemic equilibrium point analysis $(S, I, R) = (0{,}61517; 0; 0)$ and $R_0 = 0{,}00001$, it can be concluded that Leptospirosis will not become endemic in Pati with the endemic point $(S^*, I^*, R^*) = (51{,}64700; -1.647{,}48037; -32.098{,}11767)$ which is unstable and is even likely to disappear from the district.

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