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Search for Electron-Electron Interaction In Cooper Pairs In Terms of Universal Constants

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ABSTRACT

Superconductors whose properties can be described by the BCS theory are called conventional superconductors. The others are called unconventional superconductors. Formation of Cooper pairs and their frictionless flow is central to the theories of superconductivity that have been proposed so far. Simultaneously, the nature of interaction between the charges constituting the Cooper pair is an important parameter. This viewpoint is supported by the fact that the magnetic flux quantum is the same for all superconductors since it is composed of two universal constants and is due to the motion of Cooper pairs that carry the electric charge whose value is $2e$. Thus, for such an interaction to be constant, it should be composed of some universal constants, and we are yet to discover and predict this interaction. Calculations have been done using the existence of some fundamental forces between the electrons in the Cooper pair to see if such an interaction can be discovered. It is noted that the binding energy of Cooper pair depends on universal constants, except the effective mass(m_e) that may vary from superconductor to superconductor. It is possible that the variation of m_e leads to different transition temperatures T_c for various types of superconductors.

Keywords: Superfluid flow, Cooper pairs, Unconventional superconductors, Magnetic flux quantum.

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1. INTRODUCTION

Superconductivity was discovered by H. Kamerlingh Onnes in 1911 (Onnes, 1911). It is the vanishing of the electrical resistance of a material (conductors that are metallic, metallic alloys, semiconductors, and cuprates, etc.) at very low temperatures or somewhat high temperatures (T_c > 30K) resulting in the flow of very large currents of the order of 10^5 amperes. The central concept is that it is the frictionless and non-dissipative flow of charges that leads to large currents. It is found that the charges contributing to the flow of large currents come together to form pairs, called the Cooper pairs, with a finite attractive interaction (Cooper, 1956; Bardeen *et al.,* 1957).

Based on the concept of the formation of Cooper pairs in which the two electrons have opposite spins and momenta (time-reversed states) and assuming their existence near the Fermi surface, the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity was proposed (Bardeen *et al.,* 1957). This attractive interaction between a pair of electrons, called a Cooper pair, was assumed to be due to an exchange of a phonon between the two electrons constituting the pair. This interaction was called electron-phonon interaction. The number of electrons that take part in the formation of Cooper pairs is a very tiny fraction (10^{-4}) of the total number of electrons that exist in the material. The rest, 99.99% are the conduction electrons (Mourachkine, 2004). Thus, conventional BCS theory is a consequence of two main aspects of conventional superconductors: one is that the normal state is the usual metallic state, which is appropriately described by a Fermi gas of non-interacting electrons; and two is that the Cooper pairs are built from two electrons which effectively attract each other due to the phonons of the underlying lattice, and that the spins and momenta of the electrons constituting the Cooper pair are reversed, leading to S-wave pairing (total spin of the Cooper pair, S=0). In fact, Cooper (1956) demonstrated that a weak attractive interaction exists between a pair of electrons and that this paired state has an energy slightly lower than the Fermi energy. This means that the pair is bound. Simply expressed, Cooper pairing refers to the phenomenon in which two electrons, in a superconductor form a pair. These pairs have opposite spins and momenta, exhibit quantum entanglement, and move through the material without energy dissipation.

It is well known that the structure of Cooper pairs plays an important role in the flow of large superconducting currents (Kadin, 2007). A metal is a degenerate gas of electrons (fermions) piled up in the momentum-space, up to the so-called Fermi level (E_F). Since the electrons near the Fermi surface are the ones that form Cooper pairs, the last electrons entering the game of superconductivity have large energy, of the order of 10000K and these are the only accessible electrons since the ones deep inside the Fermi sea are frozen up to the top of the Fermi surface. Now, although the electrons at Fermi level have high energies - equivalent to temperatures on the order of 10000K, the situation changes when two such electrons with momenta k_F and $-k_F$, come together to form a Cooper pair. The total momentum of the pair becomes $k_F + (-k_F) = 0$, indicating a bound state at rest in the centre of mass frame.

Due to Pauli exclusion principle, the two electrons must have opposite spins, resulting in spin-singlet state, where the total spin S=0. In quantum mechanical terms, this corresponds to singlet pairing, with spin multiplicity given by 2S+1=0+1=1. It may appear puzzling that the large energy associated with electrons at Fermi level, corresponding to temperatures around 10000K seemingly disappears when cooper pairs form. This phenomenon is called Cooper instability, which refers to the fact that even an arbitrary weak attractive interaction between two electrons near the Fermi surface can form a Cooper pair.

To conserve electric charge, each Cooper pair carries a charge of 2e, since it consists of two electrons. These pairs form just outside the Fermi surface of the Fermi sea, where the pairing instability is most effective.

The electrons interact with each other through a weak attractive force which is called screened Coulomb interactions due to lattice distortion (essentially, the interactions is mediated by phonons, resulting in a net attraction between electrons that overcomes their natural Coulomb repulsion).

Another important parameter that characterizes Cooper pairs is the coherence length (ξ), which represents the average spatial separation between two electrons in a Cooper pair. It defines the distance over which the two electrons remain quantum mechanically correlated. If the inter-electron distance exceeds the coherence length (ξ), the Cooper pair breaks, and superconductivity is lost. The typical mean separation at which pairing correlation becomes effective lies between 100nm and 1000nm (where 1nm = 10^{-9}m), depending on the material. This correlation arises from an effective attractive interaction between electrons. Because the coherence length is much larger than the inter-electron spacing in a metal, many Cooper pairs can overlap significantly. In fact, within the volume occupied by one Cooper pair, thousands of other pairs may coexist, with each pair maintaining its quantum correlation. This overlapping nature is the hallmark of the quantum macroscopic coherence in conventional superconductors.

Over time, various types of superconductors have been discovered, broadly categorized into two main groups:

- Conventional superconductors, which are sufficiently described by the BCS theory, and
- Unconventional superconductors, each of which requires a different theoretical framework for explanation.

One notable class of unconventional superconductors is Heavy Fermion (HF) systems. These are inter metallic compounds that contain rare-earth or actinide elements with partially filled f-orbitals. The strong electronic correlations in these materials lead to emergence of quasi-particles with extremely large effective masses (m_{oe}) ranging from approximately $50m_{oe}$ to $100m_{oe}$ (m_{oe} is the rest mass of the electron). The first heavy fermion superconductor ($CuCe_2Si_2$) was discovered in 1978 (Steglich et al.,1978), marking a significant departure from BCS paradigm. Since then, many other heavy-fermion superconductors have been discovered (Petrovic *et al.,* 2001). In heavy fermion systems, the localized f-electrons hybridize with conduction electrons, resulting in the creation of quasi-particles with a large increase in effective mass (m_e*) of the electrons that come together to form Cooper pairs, although the underlying pairing mechanism is often believed to be non-phononic and remains an active area of research (Mathur *et al.,*1998).

Another important class of unconventional superconductors is the high-temperature superconductors (HTSC), first discovered in 1986 (Bednorz and Müller, 1986). They observed superconductivity in La-Ba-Cu-O and La-Sr-Cu systems with critical temperatures(T_c) in the range of 30-40 K. Shortly afterward, higher- T_c superconductivity was found in the Ya-Ba-Cu-O systems with T_c≈$90K$ and later in Bi-Ca-Sr-Cu-O systems where T_c exceeded 110 K. These materials, known collectively as cuprate superconductors, marked a major breakthrough because their critical temperatures far exceeded those of conventional superconductors. However, it soon became evident that the experimental behaviour of HTSC could not be explained by the conventional BCS theory (Di Castro and Bonolis, 2014; Nicol and Carbotte, 1991). There were two major departures of high-T_c from the BCS theory. First, the pairing mechanism in high-T_c superconductivity may not be due to some weak phonon-mediated interactions, as in conventional superconductors. Instead, it may arise from exotic pairing mechanism, such as those involving strong electronic correlation or spin fluctuations.

Second, some theories propose that high-T_c superconductivity may not be due to condensation of Cooper pairs but from the condensation of new quasi-particles of charge +e, known as holons. In fact, the problem of high-T_c superconductivity is very much connected to the problem of a complex electron phase diagram with a whole set of exotic ordering phenomena occurring at comparable ordering temperatures. The different ordering phenomena may be described as competing since they are fighting for the same electrons. There may be S-wave pairing, p-wave pairing, and d-wave pairing, and any two of them may exist simultaneously. But co-existence at a microscopic scale is generally quite rare, and if it occurs, one order usually acts to suppress the other. In fact, competing orders may provide the pre-requisites for the onset of superconductivity.

In the case of cuprates, we are dealing with a strange Mott insulating system La_2CuO_4 that becomes a superconductor by electron or hole doping of the CuO_2 layers. Each of the copper atoms in CuO_2 planes has an unpaired electron, resulting in an effective spin of 1/2. These spins order into a three-dimensional Néel antiferromagnetic (AFM) state. However, upon doping with electrons or holes, this AFM order disappears quickly.

Hole doping largely affects AFM order, but AFM order survives in the form of an incommensurate spin density wave order in some range of hole doping. In contrast, Electron doping is less destructive to AFM order, and there is substantial experimental evidence for the coexistence of superconductivity and antiferromagnetism in electron-doped cuprates (Armitage *et al.,* 2010; Rahman *et al.,* 2015; Mazumdar, 2018).

Moreover, both charge and spin ordering phenomena are observed in these systems, alongside the emergence of a pseudogap phase. Interestingly, in electron-doped crystals, the removal of apical oxygen atoms has been found to induce superconductivity even without doping (Rahman *et al.,* 2015; Mazumdar, 2018).

A characteristic feature of cuprate superconductors is the appearance of superconducting dome as a function of electron or hole doping. In these materials, it was established that the charge carriers responsible for superconductivity are electron pairs, as demonstrated by the flux quantization measurements (Gough *et al.,* 1987). It is well known that the quantum of magnetic flux (φ) in superconductors is due to motion of Cooper pairs that carry an electric charge 2e, and φ is the same for all types of superconductors.

Superconductivity, whether conventional or unconventional, involves two key aspects. The first is the nature of normal state. In conventional metallic systems, this state is typically described by a Fermi gas of non-interacting electrons. The second and the most crucial aspect is the formation of Cooper pairs. These pairs arise from an effective attractive interaction between electrons, allowing them to condense into a macroscopic quantum state that exhibits zero resistance and the expulsion of magnetic fields (the Meissner effect).

In unconventional superconductors such as cuprates, iron-based superconductors, heavy fermion superconductors, and spin-fluctuation-mediated superconductors, the normal state cannot be accurately described by a simple Fermi gas of non-interacting electrons. This renders the first aspect of conventional superconductivity inapplicable. Thus, the second aspect is the most relevant one, and thus the electron-electron interactions give rise to a number of different ordering phenomena, including spin-density waves, charge order, and pseudogap behavior. Therefore, any proper theoretical description of unconventional superconductivity must account for the complex underlying electronic structure and the many-body interactions at play. Understanding these interactions is essential for uncovering the mechanisms behind high-temperature superconductivity and the rich phase diagrams observed in these materials.

The problem of pairing glue is the most challenging problem of unconventional superconductivity. It looks certain that the glue has to be provided by the electrons themselves, and we are yet to predict that kind of glue or electron-electron interactions.

Another approach that characterizes both cuprates, heavy fermions, and iron pnictides is the proximity to the magnetic order. The spin-spin correlations are quantified by a magnetic susceptibility (χ_0). It was proposed that the pairing glue in the unconventional superconductors is due to the intense low-energy magnetic fluctuations present in the proximity of magnetic order. By this approach, the same mechanism that renders the system magnetic could also lead to superconductivity. This, in general, is the idea behind spin-fluctuation mediated pairing.

Keeping in mind the ideas discussed above, the minimal model that contains all the important ingredients is the Hubbard model Hamiltonian. The **Hubbard model** serves as a minimal yet powerful framework that captures the essential physics of strongly correlated electron systems, including unconventional superconductors. The Hubbard Hamiltonian is given by:

$$H=-\sum_{i,j,\sigma} t_{i,j}\, C_{i\sigma}^{+} C_{j\sigma} + U\sum_i n_{i\uparrow}\, n_{i\downarrow} - \mu\sum_{i,\sigma} n_{i\sigma} \quad (1)$$

Here $C_{i\sigma}^{+}$ creates an electron on the site i with spin σ and this can hop between neighboring sites and the next-neighbor sites with an associated energy of t and t' respectively. The interaction U denotes the energy cost associated with having two electrons on the same site, and the average electron density is controlled by the chemical potential, μ. The Coulomb interaction between the electrons provides the tendency towards magnetic order when it is quite large compared to the hopping energies, and it also brings about the superconducting glue by higher-order quantum processes. This indicates that there may exist some sort of fundamental interactions between the electrons in the Cooper pairs, one of the interactions could be Coulomb interaction and may be the glue related to gravitational attraction between the electrons or some other unknown interaction.

In a recent experimental study on Cooper pairs (Han, 2024), intriguing evidence was found suggesting the presence of exotic charge carriers in superconducting state. Specifically, the study reported that in a particular an exotic metal, the superconducting charge carriers exhibited effective charges that appear to have 4 and 6 times the charge of a single electron, suggesting the formation of Cooper pair molecules. Such findings challenge the conventional understanding of superconductivity based solely on 2e Cooper pairs and may point toward new forms of electron pairing mechanisms or emergent many-body quantum states in strongly correlated systems.

The discovery of exotic electronic behaviour in Kagome metal has significantly broadened our understanding of quantum materials. In 2019, Dirac fermions and flat bands were discovered in Kagome metal, as reported by Kang *et al.,* (2020), highlighting the potential of these materials to host strongly correlated and topologically nontrivial electronic states. Shortly thereafter, superconductivity was discovered in the Kagome metal Cesium-Vanadium-Antimonide (CsV_3Sb_5 or CVS) (Ortiz *et al.,* 2020; Berkowitz, 2020), further intensifying interest in the Kagome lattice as a platform for unconventional superconductivity.

More recently, an even more surprising phenomenon was reported: the emergence of **quasiparticles carrying electric charges multiple times that of a single electron,** including charges of 4e and 6e (Ge *et al.,* 2024). This challenges traditional views, which held that electron pairing forces become increasingly weaker as more electrons are involved, making such multi-electron bound states unlikely. These discoveries can result in uncovering new ways of superconductivity due to more new kinds of Cooper pairing.

The manifestation of Cooper pairing existence with flux quantum of $\Phi_0=\frac{h}{2e}$, was observed by William Little and Ronald Park in 1962 (Little and Parks, 1962). This flux quantization is a hallmark of conventional superconductivity, where Cooper pairs form the basic charge carriers. However, recent experimental findings have revealed striking deviations from this standard picture. In particular, Han (2024) reports the observations of oscillations at the flux period of $\Phi_0=\frac{h}{6e}$ between 2K and 3K, and a flux period of $\frac{h}{4e}$, between 1K and 2K, with the $\frac{h}{2e}$ period becoming dominant only in the zero-resistance regime below 1K. The periodicities, $\frac{h}{6e}$ and $\frac{h}{4e}$ also imply that two or three Cooper pairs may recombine to form Cooper molecules with 4e or 6e total charge. It is these Cooper molecules that may turn to be responsible for the superconductivity. Such a superconducting order will be due to the higher charge Cooper pair state in the superconductors. Hence, there may be superconductors in which Cooper pairs with charge 2e move without dissipation, and there may be superconductors in which Cooper molecules with charge 4e or 6e flow without dissipation. The existence of such states would necessitate a revision of how pairing interactions are understood in strongly correlated systems.

Furthermore, the formation of these multi-pair bound states may involve significant modification of Coulomb interactions and potentially even the gravitational interactions at microscopic scales between electrons. Motivated by this possibility, theoretical investigations have been undertaken in which these **modified interactions** (both electromagnetic and gravitational) are used to calculate the **effective electron-electron attraction** within Cooper pairs and Cooper molecules. These studies aim to determine whether the resulting interaction energies can be expressed in terms of a **set of universal physical constants,** potentially revealing deeper, fundamental principles that govern superconductivity, particularly in its unconventional forms.

2. THEORETICAL FRAMEWORK

It was experimentally proved long ago (Doll and Näbauer, 1961) that the magnetic flux quantum in superconductors is given by

$$\phi_0=\frac{h}{2e} = 2.067\times10^{-15}\,\mathrm{Wb} \tag{2}$$

where h is Planck's constant and e is the charge of the electron.

This magnetic flux quantum in superconductors is due to the motion of Cooper pairs that carry 2e electric charge, and that this is the same for all types of superconductors, conventional and unconventional (Ishiguri, 2021). It is therefore quite possible that the interaction between two electrons in the Cooper pairs should also be the same for all types of superconductors. This is important because the flux quantum in superconductors is due to the motion of Cooper pairs. The interaction between the electrons should be due to some universal constants so that it has a constant value for all types of superconductors, and for bound Cooper pairs, it must be attractive.

As a rule, nature does not allow fundamentally different types of electron-electron interactions within the Cooper pairs that are responsible for superconducting currents in all types of superconductors. After all, electrons have no eyes to determine what kind of material they are flowing through. Likewise, when they come together to form a Cooper pair, they have no means of deciding in which material they will flow. The primary role of Cooper pairs is to move without dissipation, enabling large, resistance-free currents regardless of the nature of the superconductor.

While there are conflicting views as to whether the Cooper pairs are formed before transition and after the transition, this distinction in not essential for the present study. Instead, the focus here is to investigate whether the effective interaction between electrons in a Cooper pair can be derived from a combination of universal constants, such as Planck's constant *(h)*, electron charge *(e)*, and electron rest mass (m_e) or some other universal constants.

The primary objective of this study is to explore whether the interactions between the electrons in a Cooper pair can be described in terms of fundamental energies such as, Coulomb energy, gravitational energy, and possibly some unknown potential energy, under the condition that the net interaction between the electrons must be attractive for a stable Cooper pair to form.

To simplify the analysis, a sweeping assumption is made such that there exists only Coulomb interaction and the gravitational interaction between the electrons in the Cooper pair. In this framework, the gravitational attraction, though typically negligible at atomic scales, is hypothesized to be slightly stronger than the repulsive Coulomb interaction. This speculative scenario allows for the derivation of **critical conditions** by equating the magnitudes of gravitational and Coulomb potentials between the electrons. Specifically, when the repulsive Coulomb interaction is exactly balanced by the attractive gravitational interaction, it defines a threshold condition under which a bound Cooper pair may form. By solving this equality, one can obtain critical values for the physical parameter involved, expressed in terms of universal constants.

To initiate this exploration, we adopt a **first-order approximation** using **long-range classical formulas** for the two interactions. While highly idealized, this approach serves as a conceptual starting point to investigate whether a **universal interaction framework** (based solely on known forces and constants) can plausibly account for electron pairing in superconductors, irrespective of material-specific complexities.

If F_{el} is the electrostatic force (interaction) between the two electrons in the Cooper pair, then:

$$F_{el}=\frac{e^2}{4\pi\epsilon_0 r^2} \qquad (3)$$

where e=1.6×10^{-19} Coulombs is the charge of the electron, ϵ_0=8.85×10^{-12}F/m is electric permittivity, r= 1000Å, is the distance between the two electrons in the Cooper pair.

The size of the Cooper pair is 10^3-10^{4}Å for conventional (BCS) type superconductors, and it is 1-10 Å for unconventional superconductors.

If F_{gr} is the gravitational interaction between the two electrons in the Cooper pair, then

$$F_{gr}=G\frac{(m_e)^2}{r^2} \qquad (4)$$

where G=6.6×10^{-11}Nm^2kg^{-2} is the universal gravitational constant.

3. RESULTS AND DISCUSSIONS

For the two forces to be equal, i.e., $F_{el}=F_{gr}$, we will get,

$$\frac{e}{m_e}=(4\pi\epsilon_0 G)^{\frac{1}{2}} = 0.86\times10^{-10}\ \text{C/kg}. \tag{5}$$

From Eq. (5), the effective mass of the electron will be,

$$m_e= \frac{e}{0.86\times10^{-10} C/kg}=1.86\times10^{-9} kg \tag{6}$$

The value of the effective mass of the electron obtained in Equation (6) is very large compared to the rest mass of the electron, such that

$$m_e=2.04\times10^{21} m_0 \tag{7}$$

where $m_0=9.1\times10^{-31} kg$ is the rest mass of the electron.

Thus, for the net interaction to be slightly attractive, the effective mass of the electron should be more than $2.04\times10^{21} m_0$. Although there are heavy fermion superconductors in which m_e lies between 100 m_0 to 1000 m_0 or so, but this is extremely high and may be unacceptable unless we decide that in the superconducting state, extremely heavy mass electrons ride side by side as charged wire carrying huge superconducting currents (Mishonov, 1994). For the net attractive interaction, denoted by E_0, between the electrons in the Cooper pairs, we can write:

$$E_0=\frac{G(m_e)^2}{r}-\frac{e^2}{4\pi\epsilon_0 r} \tag{8}$$

Now due to the extremely large effective mass of the electrons, it is assumed that the electrons sit side by side as a charged wire. Then the value of r will be twice the radius of the electron

$$r=2r_e= 5.636\times10^{-15}\ \text{m} \tag{9}$$

Equation (8) can now be written as:

$$E_0=\frac{G(m_e)^2}{2r_e}-\frac{e^2}{4\pi\epsilon_0(2r_e)} \tag{10}$$

Equation (10) is the binding energy of a pair of electrons in a Cooper pair, and it is in terms of the universal constants G, m_e, e, r_e, π and ϵ_0. These Cooper pairs effectively behave as bosons and flow without dissipation. Since they are so close to touch each other, an assembly of such pairs may resemble a charged wire. In Equation (10), E_0 is the difference between the Coulomb repulsion between the two electrons and the attractive gravitational energy between the electrons. This energy depends on a universal constants, except the effective mass (m_e), which may vary from one superconductor to another. When m_e is expressed in terms of the electron's rest mass m_0, it becomes a universal constant; however, its value differs across different superconductors.

So far, there has been no experimental observation that can confirm such a picture. However, a study by Mishonov (1994) presents experimental evidence suggesting that like-charged particles can attract each other. This phenomenon may also be relevant to Cooper pairs, in which the bound particles are electrons.

Another recent experimental observation by Arfan (2024) suggests that Cooper pairs may, under certain conditions, coalesce into Cooper molecules with charge 4e and or 6e. Due to proximity and short spatial distances and short dimensions involved, such configurations may give rise to novel quasi-particles with extraordinary high effective masses, on the order of $10^{16} GeV \left(1G=10^{9}\right)$. For comparison, the Planck scale is approximately $10^{19} GeV$, highlighting the potential significance of these findings.

Thus, if we write Planck's energy as:

$$E=mc^{2}=10^{19}\times10^{9}\times1.6\times10^{-12} erg \tag{11}$$

Then the corresponding mass is:

$$m=\frac{1.6\times10^{16}}{9\times10^{20}}=0.1777\times10^{-4} g \tag{12}$$

Now, the rest mass of the electron $m_0=9.1\times10^{-28} g$. Hence, the effective mass of the electron is:

$$m_e=\frac{m}{9.1\times10^{-28}}=1.95\times10^{22} m_0 \tag{13}$$

This value is close to the value given in Equation (7) using the classical gravitational and electromagnetic constants.

In another approach, if we consider gravity manifesting directly at short distances and adopt an **electroweak-scale gravitational constant** given by:

$$G_e=2.909\times10^{22} \frac{m^3}{kg.s^2}\left(\frac{Nm^2}{kg^2}\right) \tag{14}$$

Then, from Equation 5, the electron mass m_e can be estimated as:

$$m_e=\frac{e}{(4\pi\epsilon_0 G_e)^{\frac{1}{2}}}=0.1779\times10^{-24} kg=1.9549\times10^{5} m_0 \tag{15}$$

This value is much smaller than the value given in Equation (13), and hence it is more acceptable, but it is still quite large compared to the heavy fermion values of $100m_0$ to $1000m_0$ that are known so far.

Now, when we calculate the values of the Coulomb energy E_C and the gravitational energy E_g between the electrons in a Cooper pairs, using m_e=1.955 ×$10^5 m_0$ and r=1 nm , we obtain:

$$E_C=2.3\times10^{-19}J \tag{16}$$

and

$$E_g=9.458\times10^{-29}J \tag{17}$$

These results show that even with a significantly increased effective mass m_e, the gravitational interaction remains many orders of magnitude weaker than the Coulomb repulsion at the nanometer scale.

For a bound Cooper pair, the interaction energy between the electrons in the Cooper pair must be net attractive and slightly greater than the repulsive Coulomb energy. This condition requires that the effective mass m_e be greater than the value obtained in Equation (15). By comparing the values of E_C and E_g given in Equations (16) and (17), we find that the effective mass m_e must be approximately 10^5 times **larger** than the value given in Equation (15). Since $E_g \propto m^2$, increasing effective mass m_e would significantly make gravitational attraction stronger. Thus, the effective electron mass required for such binding is:

$$m_e \cong 1.9549\times10^{10} m_0 \tag{18}$$

This value is small compared to the values obtained in Equations (7) and (13), but it is still significantly larger than the known values of the effective masses m_e of the electrons in the Cooper pairs. A graphical illustration of effective masses of Cooper pairs in terms of electron rest masses is shown in Figure 1.

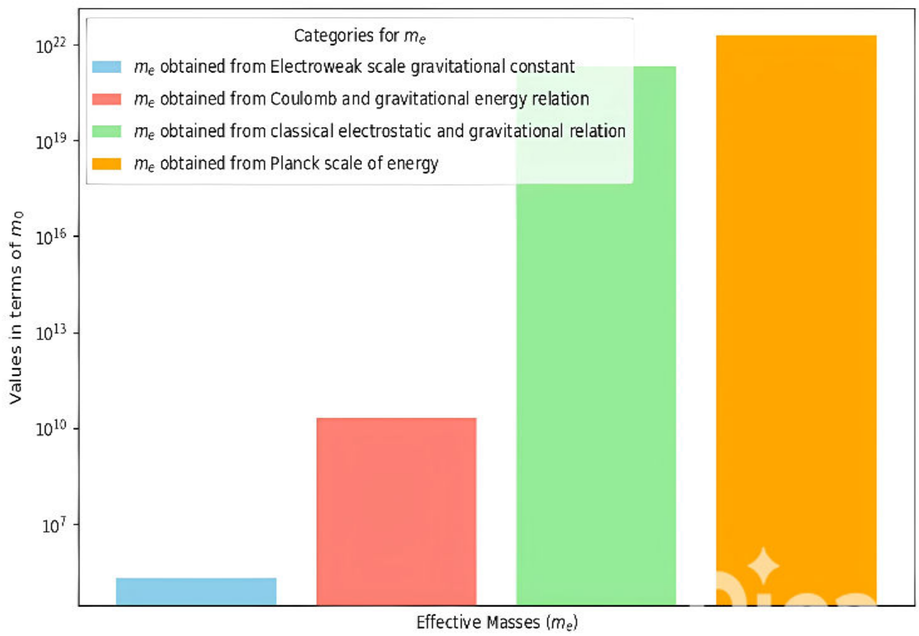

Figure 1. Bar graph of Effective masses

4. CONCLUSIONS

This study theoretically investigated the possibility that gravitational interactions could contribute to the binding of electrons in Cooper pairs, a central mechanism in superconductivity. By comparing gravitational and Coulomb forces at nanoscales, it is shown that achieving a net attractive interaction would require extremely large effective electron masses ranging from $10^5 m_0$ to $10^{22} m_0$ far exceeding those observed in known superconductors. While these values appear physically extreme, theoretical considerations and recent experimental suggestions (Mishonov, 1994; Arfan, 2024) raise the possibility that under certain conditions, such ultra-heavy quasiparticles might form, potentially resembling tightly bound "charged wires" or Cooper molecules.

Based on our calculations, it is possible that variations in the effective electron mass contributes to the different transition temperatures T_C observed across different superconducting materials. A recent experimental observation of Cooper molecules with charges 4e and 6e further highlight the complexity of the superconducting state. The nature of the electron pairing whether S-wave singlet, triplet, or involving higher angular momentum states like p-wave or d-wave is influenced by spin alignment and local magnetic interactions, which may also serve as the "glue" for pairing via spin fluctuations.

To develop a comprehensive theory of superconductivity that incorporates these diverse pairing mechanisms, spin configurations, and mass variations is immensely challenging. However, any successful theory must be firmly rooted in universal constants, adhere to fundamental principles of physics, and discard unphysical assumptions. The key lies in identifying the essential physical principles that enable Cooper molecule formation and accurately predicting key parameters such as T_C. Constructing such a theory will be the focus of our future research efforts.

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