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ON SIGNED-DIGIT NUMBERS

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ABSTRACT

Signed digit numbers are defined as the numbers where each individual digit may have positive or negative numbers. Properties of the signed digit numbers are investigated in detail. New definitions associated with the signed-digit numbers are given. Theorems are posed and proven for the signed digit numbers. Special prime numbers associated to the signed digit numbers are given. Numbers in different bases are also treated. Divisibility rules are given for such numbers.

Keywords: Integers, Prime Numbers, Divisibility, Negative Digits.

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1. INTRODUCTION

In a conventional integer written in base 10, all digits may contain ten different numbers ranging from 0 to 9. If one allows the digits to be negative also, then all digits may contain 19 numbers ranging from -9 to 9. The new system is called the signed digit numbers. One of the deficiencies of the signed digit numbers is their redundancy, that is, there are many ways of expressing such numbers corresponding to the same numerical value. However, the advantages go far beyond this. It is shown that signed digit numbers lead to faster and more efficient computational algorithms requiring less memorial capacity [1-7]. There are advantages of using signed-digit numbers for image processing also [8]. In our daily lives, we encounter with simple problems that can be expressed in terms of signed-digit numbers. Assume that we are on a shopping and want to buy a shoe with price 87$. We give a cash of 100$ and get a return of 13$. In terms of signed digit numbers, the price is $1\bar{1}\bar{3}$$ which is equivalent to 87$. To express the negative sign of the digit, usually an overbar is used. A daily life weight problem was solved by employing base 3 signed-digit numbers [9].

In this work, a detailed analysis of the signed-digit numbers (SDN) are presented. The paper is organized as follows. In Section 2, the basic definitions and theorems reflecting the properties of such numbers are given. In Section 3, numbers with different bases than the conventional base 10 are considered. In Section 4, some new classes of SDN are proposed with new names given to them. In Section 5, prime numbers are treated in association with the SDN. Finally, in Section 6, the divisibility rules are given for such numbers. A summary is given in the concluding remarks section with directions for further work.

2. BASIC DEFINITIONS AND THEOREMS

Definition 1. An *n*+1-digit signed-digit number (SDN) is expressed as $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0$ where $a_i \in \{-9,-8,\dots,-1,0,1,\dots,8,9\}, i = 0,1,2,\dots n$ □

When writing the specific numbers, the negative digits may be expressed with overbars to distinguish them from the positive digits. For example, $1\bar{3}7\bar{2}$ is a signed digit number. A negative integer is indeed a signed digit number with all its digits being negative numbers, i.e. $-321 = \bar{3}\bar{2}\bar{1}$. In the signed digit system, there is no need to put a sign in front of the number.

Definition 2. The numerical value of a signed digit number written in base 10 is

$a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0 = \sum_{i=0}^{n} a_i 10^i$ where $a_i \in \{-9,-8,\dots,-1,0,1,\dots,8,9\}, i = 0,1,2,\dots n$ □

Indeed, the numerical value of the signed digit number will correspond to a number having positive digit numbers (PDN). The conventional integers are all PDN having plus or minus sign in front. Zero is an exception to this rule since $\bar{0} = 0$.

Definition 3. The magnitude of a SDN $|a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0|$ is defined as $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0$ where $a_i \in \{0,1,\dots,8,9\}, i = 0,1,2,\dots n$ □

Hence $|2\bar{7}8\bar{3}| = 2783$.

One of the major deficiencies of SDN is their non-uniqueness property. For example,

$11\bar{9} = 2\bar{9}\bar{9} = 101$ or $15\bar{7} = 2\bar{6}3 = 143$.

However, PDN's have a unique representation with a unique numerical value [10].

Theorem 1. Among the numerically equivalent SDN's, the least in magnitude corresponds to that of the equivalent PDN □

Proof. For numerical values corresponding to a positive integer, since the SDN's at least have one negative digits, a subtraction occurs in evaluating the numerical value which corresponds to the PDN. Hence the magnitude of the PDN is always less than or equal to the magnitude of the equivalent SDN. The same is true for numerical values corresponding to a negative integer. In that case, there is at least one positive digit which leads to a reduction again □

For numbers with positive numerical values such as 101, it is evident that $|101| = 101 < |11\bar{9}| = 119 < |2\bar{9}\bar{9}| = 299$. For numbers with negative numerical values such as -445, $|-445| = 445 < |\bar{4}\bar{5}5| = 455 < |\bar{5}55| = 555$.

Definition 4. For a given PDN, the least equivalent SDN is defined to be the one having the same numerical value with the least magnitude among the many SDN □

For the number 101, then the least equivalent SDN is $11\bar{9}$ and for the number -445, the least equivalent SDN is $\bar{4}\bar{5}5$.

The elementary arithmetic operations of addition, subtraction, and multiplication can be done similar to the PDN taking into account the signs. For example:

$$5\bar{6} + 2\bar{4} = 60, \quad 2\bar{7} + 1\bar{1} = 3\bar{8}$$

$$7\bar{8} - 4\bar{3} = 3\bar{5}, \quad 1\bar{4}1 - 9\bar{6} = \bar{3}7$$

$$3\bar{6}x1\bar{2} = 2\bar{1}2, \quad 1\bar{2}3x3\bar{9} = 2\bar{3}5\bar{7}$$

The division is not so straightforward, however, and the usual division process of PDN should be applied with extensive care.

$$20\bar{8}/3\bar{6} = 1\bar{2}, \quad 77\bar{5}/2\bar{5} = 6\bar{9}$$

Theorem 2. For a given *n*+1-digit arbitrary SDN $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0$, if $a_n \geq 2$, then the equivalent PDN is also an *n*+1 digit number $b_n b_{n-1} b_{n-2} \dots b_2 b_1 b_0$. If $a_n = 1$, then the equivalent PDN may consist of any number of digits *m* with $1 \leq m \leq n + 1$□

Proof. The proof is straightforward and left as an exercise □

For three-digit SDN's, if the highest digit is 1, then there are alternatives of 1,2 and 3-digit equivalent PDN's. For example:

$$1\bar{9}\bar{9} = 1, \quad 1\bar{2}1 = 81, \quad 19\bar{1} = 189$$

However, for a three-digit SDN with $a_n \geq 2$, the equivalent SDN is also a 3-digit number.

$$2\bar{9}\bar{9} = 101, \quad 5\bar{2}4 = 484, \quad 99\bar{2} = 988$$

Theorem 3. For a given *n*+1-digit SDN, the sign of the numerical value is determined by the sign of the highest digit term □

Proof. The numerical total of all digits other than the highest digit cannot be larger than the numerical value of the highest digit, and hence the sign of the numerical value is determined by the highest digit term □

As examples, $3\bar{8}\bar{9} = 211$, $\bar{1}99 = -1$

Theorem 4. For a given *n*+1-digit PDN, infinitely many *m*+1-digit SDN can be written with $m > n$ □

Proof. An algorithm may be constructed to produce the equivalent SDN's with higher number of digits. For the given PDN $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0$, write an equivalent one-digit higher SDN $1\bar{b}_n \bar{b}_{n-1} \bar{b}_{n-2} \dots \bar{b}_2 \bar{b}_1 \bar{b}_0 = a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0$, calculate $b_n b_{n-1} b_{n-2} \dots b_2 b_1 b_0$ and add $\bar{9}$ as many as you wish after the highest digit 1 □

As an example, take the number 58. Then $58 = 100 - b_1 b_0$ or $b_1 b_0 = 42$. The infinitely many higher digit SDN's can be constructed as follows

$$58 = 1\bar{4}\bar{2} = 1\bar{9}\bar{4}\bar{2} = 1\bar{9}\bar{9}\bar{4}\bar{2} = 1\bar{9}\bar{9}\bar{9}\bar{4}\bar{2} = \cdots$$

Theorem 5. For an *n*-digit PDN, the total possible numbers are $18x10^{n-1}$and for an *n*-digit SDN, the total possible numbers are $18(19^{n-1} - 10^{n-1})$ excluding the PDN's □

Proof. For a positive PDN**,** the highest digit has 9 options and all other digits have 10 options. Multiplication of all options yield $9x10^{n-1}$. If the negative integers are also included, then the total number is $2x9x10^{n-1}$. For SDN's the highest digit has 18 options excluding 0 and the remaining digits have 19 options. Hence the total number is $18x19^{n-1}$ and subtracting the PDN's from this, the total number of SDN's become $18x19^{n-1} - 2x9x10^{n-1} = 18(19^{n-1} - 10^{n-1})$ □

If *n*=2, the number of PDN is 180 and the number of SDN excluding the PDN is 162. If *n*=3, the number of PDN is 1800 and the number of SDN excluding the PDN is 4698.

Theorem 6. The number 0 is uniquely represented in the signed-digit system □

Proof. Assume that an *n*+1-digit SDN is numerically equal to zero

$a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0 = a_n 10^n + a_{n-1} 10^{n-1} + \cdots + a_2 10^2 + a_1 10 + a_0 = 0$. Since the order of magnitudes are different from each other and cannot be balanced to yield zero, i.e. $abs(a_n 10^n) > abs(a_{n-1} 10^{n-1} + \cdots + a_2 10^2 + a_1 10 + a_0)$, the number should be a single digit number. But among the single digit numbers, only $\bar{0}$ has a numerical value of 0. Hence the representation $\bar{0} = 0$ is unique □

Theorem 7. For a given PDN, the sum of the PDN and all its possible SDN's with similar digits are zero □

Proof. Proofs are given for two and three-digit numbers. Higher digit proofs follow by induction. For two-digit numbers, the possible sum is

$$ab + a\bar{b} + \bar{a}b + \bar{a}\bar{b} = ab + a\bar{b} - a\bar{b} - ab = 0$$

and for the three-digit numbers, the possible sum is

$$abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + \bar{a}\bar{b}c + \bar{a}\bar{b}\bar{c}$$

$$= abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} - a\bar{b}\bar{c} - a\bar{b}c - ab\bar{c} - abc = 0$$ □

3. NUMBERS IN OTHER BASES

Similar definitions and theorems apply for SDN's written in bases other than 10.

Definition 5. An *n*+1 digit signed digit number in base *r* is expressed as $(a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0)_r$ where $a_i \in \{-(r-1), -(r-2), \dots, -1,0,1, \dots, r-2, r-1\}, i = 0,1,2, \dots n$ □

Definition 6. The numerical value of a signed digit number written in base *r* is

$(a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0)_r = \sum_{i=0}^{n} a_i r^i$ where

$a_i \in \{-(r-1), -(r-2), \dots, -1,0,1, \dots, r-2, r-1\}, i = 0,1,2, \dots n$ □

The counterparts of the other definitions and theorems given in the previous section can be written in a similar manner which will not be repeated for brevity. Some examples for such numbers are

$(10\bar{1}1)_2 = 2^3 - 2 + 1 = 7$

$(2\bar{1}\bar{1}0)_3 = 2x3^3 - 3^2 - 3 = 42$

$(6\bar{4}9)_7 = 6x7^2 - 4x7 + 9 = 275$

Note the following theorem for base-*r* numbers

Theorem 8. For a SDN with base *r* in the form of $(1a_{n-1}a_{n-2} \dots a_1 a_0)_r$ insertion of the number $\overline{r-1}$ arbitrary times after the highest digit 1 does not alter the numerical value of the SDN □

Proof. The numerical values of

$r^n = r^{n+1} - (r-1)r^n = r^{n+2} - (r-1)r^{n+1} - (r-1)r^n = \cdots$

are all equal and the numerical value of the SDN remains unchanged □

As an example, the numerical values of the numbers in base 2 are all equal

$(10\bar{1}0)_2 = (1\bar{1}0\bar{1}0)_2 = (1\bar{1}\bar{1}0\bar{1}0)_2 = \cdots = 6$

As a degenerate case of Theorem 8, the arbitrary number of inclusions of $\overline{r-1}$ after 1

$$(1\overline{r-1}\,\overline{r-1}\,\overline{r-1} \dots \overline{r-1})_r = 1$$

does not alter the numerical value of unity.

The numbers in base 2 are extremely important because they constitute the bases for computer calculations. A signed digit number in base 2 may decrease the bytes used in representing the numbers and letters and hence decrease required memory. The computational algebra becomes faster for SDN. For detailed discussions and comments on the efficiency of SDN on computer algebra, see the comments in [1-8].

4. SPECIAL SDN DEFINITIONS

Some further definitions of SDN with special properties are given in this section.

Definition 7. If the numerical value of a SDN is total square and its digital sum is also another total square, then the number is called double square SDN □

Example 1. List all two-digit double square SDN's.

The possible cases are listed in Table 1.

Table 1. Two-digit double square SDN's.

Numerical Value	SDN	Digital Sum	Check
9	$1\bar{1}$	0	Yes
16	$2\bar{4}$	-2	No
25	$3\bar{5}$	-2	No
36	$4\bar{4}$	0	Yes
49	$5\bar{1}$	4	Yes
64	$7\bar{6}$	1	Yes
81	$9\bar{9}$	0	Yes

Hence, $1\bar{1}$, $4\bar{4}$, $5\bar{1}$, $7\bar{6}$ and $9\bar{9}$ are the two-digit double square SDN's.

Definition 8. If the sum of digits of a SDN is zero, the number is called digital zero SDN □

Some sample digital zero SDN's are $4\bar{4}$, $2\bar{7}5$, $\bar{3}5\bar{2}$, $81\bar{9}$.

Definition 9. If all the digits remain the same together with their signs after division to its digital sum, the number is called sign stable SDN. If the digits remain the same with all signs changed, after division, then it is called sign exchange SDN □

Two-Digit Sign Stable SDN's: $1\bar{0}$, $2\bar{1}$, $3\bar{2}$, $4\bar{3}$, $5\bar{4}$, $6\bar{5}$, $7\bar{6}$, $8\bar{7}$, $9\bar{8}$

Two-Digit Sign Exchange SDN's: $1\bar{2}$, $2\bar{3}$, $3\bar{4}$, $4\bar{5}$, $5\bar{6}$, $6\bar{7}$, $7\bar{8}$, $8\bar{9}$

Example 2. List all three-digit Sign Exchange SDN's of the form $a\bar{b}c$.

There are 36 numbers of this form: $1\bar{2}0$, $1\bar{3}1$, $1\bar{4}2$, $1\bar{5}3$, $1\bar{6}4$, $1\bar{7}5$, $1\bar{8}6$, $1\bar{9}7$, $2\bar{3}0$, $2\bar{4}1$, $2\bar{5}2$, $2\bar{6}3$, $2\bar{7}4$, $2\bar{8}5$, $2\bar{9}6$, $3\bar{4}0$, $3\bar{5}1$, $3\bar{6}2$, $3\bar{7}3$, $3\bar{8}4$, $3\bar{9}5$, $4\bar{5}0$, $4\bar{6}1$, $4\bar{7}2$, $4\bar{8}3$, $4\bar{9}4$, $5\bar{6}0$, $5\bar{7}1$, $5\bar{8}2$, $5\bar{9}3$, $6\bar{7}0$, $6\bar{8}1$, $6\bar{9}2$, $7\bar{8}0$, $7\bar{9}1$, $8\bar{9}0$.

Definition 10. If the sum of absolute values of the digits is numerically equal to the SDN itself, the number is called Absolute Digital SDN □

Example 3. For a SDN of the form $a\bar{b}$, find the Absolute Digital SDN.

Solution. $10a - b = a + b \rightarrow \; 9a = 2b$. The only possibility is $a = 2, b = 9$ and hence the solution is $2\bar{9}$.

Example 4. For a SDN of the form $a\bar{b}c$, find the Absolute Digital SDN.

Solution. $100a - 10b + c = a + b + c \rightarrow \; 9a = b$. The only possibility is $a = 1, b = 9$ with c remaining arbitrary. Hence, the solutions are $1\bar{9}0$, $1\bar{9}1$, $1\bar{9}2$, $1\bar{9}3$, $1\bar{9}4$, $1\bar{9}5$, $1\bar{9}6$, $1\bar{9}7$, $1\bar{9}8$, $1\bar{9}9$.

5. SPECIAL PRIME NUMBERS

In this section, the prime numbers are interpreted within the context of SDN's. List of prime numbers up to the number 10^{12} is given in [11].

5.1. Carol Prime Numbers

The Carol prime numbers are defined by the formula

$$(P_c)_n = (2^n - 1)^2 - 2, \qquad n \geq 2$$

To see the similarity with SDN numbers, the formula is written in open form

$$(P_c)_n = 2^{2n} - 2^{n+1} - 1, \qquad n \geq 2$$

Therefore, the Carol Prime numbers can be written as SDN's in base 2

$$(P_c)_n = (100..\bar{1} \ldots \bar{1})_2$$

where 1 is the highest digit which is the $2n$'th digit, the intermediate digit $\bar{1}$ is the n+1'th digit and the lowest digit is $\bar{1}$. The first four Carol Primes and their equivalent SDN's are given in Table 2.

Table 2. Carol Prime Numbers.

n	Prime Number	SDN Equivalent
2	7	$(1\bar{1}00\bar{1})_2$
3	47	$(10\bar{1}000\bar{1})_2$
4	223	$(100\bar{1}0000\bar{1})_2$
5	959	$(1000\bar{1}00000\bar{1})_2$

5.2. Order of Magnitudes

Order of magnitude concept is defined below in association with the SDN.

Definition 11. For a given prime number, among the possible numerically equivalent SDN's, the least number of digits of the magnitude which is also prime is called the order of magnitude of the prime number.

Theorem 9. The order of magnitude of a prime SDN may be at least 2 and at most infinity □

Proof. See the below example □

Example 5. Determine the order of magnitudes of the prime numbers up to 113.

Solution. Results are given in Table 3. The procedure is as follows. The prime number is written as a SDN starting from the least digits. Infinitely many SDN's can be written for a given number (See Theorem 4).

The magnitudes of the SDN (See Definition 3) are checked to be prime numbers and the least number of digits for this to occur is recorded as the order of magnitude.

Table 3. Order of Magnitudes of Prime Numbers.

Prime Number	Equivalent SDN up to Prime Magnitudes	Prime Magnitude	Order of magnitude
2	$1\bar{8} = 1\bar{9}\bar{8} = 1\bar{9}\bar{9}\bar{8} = \cdots$	None	Infinity
3	$1\bar{7}$	17	2
5	$1\bar{5} = 1\bar{9}\bar{5} = 1\bar{9}\bar{9}\bar{5} = \cdots$	None	Infinity
7	$1\bar{3}$	13	2
11	$2\bar{9}$	29	2
13	$2\bar{7} = 1\bar{8}\bar{7} = 1\bar{9}\bar{8}\bar{7}$	1987	4
17	$2\bar{3}$	23	2
19	$2\bar{1} = 18\bar{1}$	181	3
23	$3\bar{7}$	37	2
29	$3\bar{1}$	31	2
31	$4\bar{9} = 1\bar{6}\bar{9} = 1\bar{9}\bar{6}\bar{9} = 1\bar{9}\bar{9}\bar{6}\bar{9}$ $= 1\bar{9}\bar{9}\bar{9}\bar{6}\bar{9} = 1\bar{9}\bar{9}\bar{9}\bar{9}\bar{6}\bar{9}$	1999969	7
37	$4\bar{3}$	43	2
41	$5\bar{9}$	59	2
43	$5\bar{7} = 15\bar{7}$	157	3
47	$5\bar{3}$	53	2
53	$6\bar{7}$	67	2
59	$6\bar{1}$	61	2
61	$7\bar{9}$	79	2
67	$7\bar{3}$	73	2
71	$8\bar{9}$	89	2
73	$8\bar{7} = 12\bar{7}$	127	3
79	$8\bar{1} = 1\bar{2}\bar{1} = 1\bar{9}\bar{2}\bar{1} = 1\bar{9}\bar{9}\bar{2}\bar{1} = 1\bar{9}\bar{9}\bar{9}\bar{2}\bar{1}$	199921	6
83	$9\bar{7}$	97	2
89	$9\bar{1} = 1\bar{1}\bar{1} = 1\bar{9}\bar{1}\bar{1} = 1\bar{9}\bar{9}\bar{1}\bar{1} = \cdots$	None	Infinity
97	$10\bar{3}$	103	3
101	$11\bar{9} = 1\bar{9}1\bar{9} = 1\bar{9}\bar{9}1\bar{9}$	19919	5
103	$11\bar{7} = 1\bar{9}1\bar{7} = 1\bar{9}\bar{9}1\bar{7} = \cdots$	None	Infinity
107	$11\bar{3}$	113	3
109	$11\bar{1} = 1\bar{9}1\bar{1} = 1\bar{9}\bar{9}1\bar{1} = \cdots$	None	Infinity
113	$12\bar{7}$	127	3

If the digital sum is multiples of 3, it is impossible to change the divisibility of 3 by successive addition of 9's. Hence, the order of magnitude becomes infinity for these cases. Excluding the first two infinity cases, the rest possess the mentioned property.

6. DIVISIBILITY RULES OF SDN

The divisibility rules of SDN can be derived similar to the PDN. Divisibility rules of 3, 4 and 7 will be given as examples.

Theorem 10. For a given *n*+1-digit SDN, if the number is divisible by 3, then

$\sum_{i=0}^{n} a_i = 3k, \quad k \in \boldsymbol{Z}$ □

Proof. $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0 = \sum_{i=0}^{n} a_i 10^i = \sum_{i=0}^{n} a_i (10^i - 1) + a_i$. Since $10^i - 1$ is always divisible by 3, then the remaining term should be divisible by three verifying the theorem □

Theorem 11. For a given *n*+1-digit SDN, if the number is divisible by 4, then

$2a_1 + a_0 = 4k, \ k \in \boldsymbol{Z}$ □

Proof. $a_n a_{n-1} a_{n-2} \dots a_2 a_1 a_0 = \sum_{i=0}^{n} a_i 10^i = a_0 + 2a_1 + 8a_1 + \sum_{i=2}^{n} a_i 10^i$. Since $8a_1 + \sum_{i=2}^{n} a_i 10^i$ is always divisible by 4, then the remaining term $a_0 + 2a_1$ should be divisible by four verifying the theorem □

Theorem 12. For a given *n*+1-digit SDN, if the number is divisible by 7, then

$\sum_{i=1}^{int(n/3+1)} (-1)^{i+1} (a_{3i-3} + 3a_{3i-2} + 2a_{3i-1}) = 7k, \ k \in \boldsymbol{Z}$ □

Proof. A proof will be given for a 6-digit number. The proof for higher digit numbers may be derived by induction.

$a_5 a_4 a_3 a_2 a_1 a_0 = (10^5 + 2)a_5 + (10^4 + 3)a_4 + (10^3 + 1)a_3 + (10^2 - 2)a_2$

$+(10 - 3)a_1 + (1 - 1)a_0 - 2a_5 - 3a_4 - a_3 + 2a_2 + 3a_1 + a_0$. Since

$(10^5 + 2)a_5 + (10^4 + 3)a_4 + (10^3 + 1)a_3 + (10^2 - 2)a_2 + (10 - 3)a_1 + (1 - 1)a_0$ is always divisible by 7, the divisibility depends on the remaining terms $-2a_5 - 3a_4 - a_3 + 2a_2 + 3a_1 + a_0$ which verifies the theorem for 6-digit numbers. One can see that there is a three-term pattern repeating itself with sign change □

In fact, these rules are the same with that of PDN, and hence any rule of divisibility for PDN remains valid for SDN also, keeping in mind that the digits may have negative signs and they should be taken into consideration.

Example 6. Check whether $485\bar{1}\bar{6}$ and $\bar{5}24\bar{1}3$ are divisible by 7.

Solution. For $485\bar{1}\bar{6}$, $\bar{6} + 3x\bar{1} + 2x5 - 8 - 3x4 = -19 \neq 7k$, hence not divisible by 7.

For $\bar{5}24\bar{1}3$, $3 + 3x\bar{1} + 2x4 - 2 - 3x\bar{5} = 21 = 7k$, hence divisible by 7.

Concluding Remarks

Signed-digit numbers (SDN) are investigated in detail. First, the basic properties are given by definitions and theorems. Numbers other than base 10 are discussed. Then some special SDN's are defined. Some applications to prime numbers are given. Finally, the divisibility rules are discussed for such numbers. SDN's can be extended to decimal, rational, irrational, and complex numbers also.

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