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Role of Cooper Pair Molecules in Determining Specific Heat and Transition Temperature of Kagome Metals in the Superconducting State.

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ABSTRACT

Superconductivity was observed in the Kagome metals cesium vanadium antimonide (CsV_3Sb_5) or CVS at the critical transition temperature T_c = 2.5K. But the resistivity of the CVS started decreasing even around 4K, and vanished at 1K. In general, for superconductors, the resistivity suddenly drops to zero at T_c. In Kagome metals, such a gradual drop in resistivity starting well above the bulk transition temperature (T_c) was observed in thin superconducting compound signifying existence of a fluctuating regime of superconductivity where Cooper pairs may be present in a disorganized form. In general, charge-2e Cooper pairs should persist from 4K (pre-formation of Cooper pairs before T_c) to T_c and below, and should give rise to $\frac{h}{2e}$ ($\frac{h}{2e} = \phi_o$, Quantized flux) resistance oscillations. Instead, the experimental observations led to the existence of oscillations at the flux period of $\frac{h}{6e}$ between 2K and 3K; and of $\frac{h}{4e}$ between 1K and 2K; and the $\frac{h}{2e}$ period becoming dominant in the zero-resistance regime below 1K only.

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Thus, the periodicities $\frac{h}{4e}$ and $\frac{h}{6e}$ could emphatically mean that two $\frac{h}{4e}$ and/or three $\frac{h}{6e}$ Cooper pairs may coalesce into Cooper molecules with Cooper molecules having a total charge of 4e and/or 6e.

Since a Cooper pair behaves effectively as a boson, these Cooper molecules can be treated as bosons, and a theory is presented, which describes the condensed state of these bosons. An expression is obtained for the quasi-particle energy in the condensed state, and this is used to calculate the specific heat (C_v), and the transition temperature (T_c). Charges of Cooper molecules will determine the magnitudes and variation of C_v and T_c with T.

Keywords: Kagome metals, Quantum Flux, Flux Periodicities, Cooper molecules, Condensed state.

1. INTRODUCTION

Superconductivity appears at some critical transition temperature T_c at which resistivity of the material becomes zero. Very large currents, of the order of 10^5A flow; two electrons combine to form pairs of electrons called Cooper pairs, effectively doubling the charge of the electric charge carriers in the superconducting state [1,2]. This is roughly the story of all types of superconductors; conventional and unconventional.

So far, the magnetic flux quantum $\Phi_o = \frac{h}{2e} = 2.067 \times 10^{-15} Wb$ is the same for all types of superconductors, and the magnetic flux quantum is due to the motion of Cooper pairs that carry an electric charge of 2e. It is still not exactly known as to what is the specific interaction between the charges in the Cooper pair, and how two similar charges form a bound attractive system. As a rule, nature must have devised the same type of interaction between charges constituting the Cooper pair for all types of superconductors. Thus, since Φ_0 is a universal constant being composed of two universal constants, the interaction between the charges in the Cooper pair should also be a universal constant since Φ_0 is a consequence of the motion of Cooper pairs.

We get flux quantization when a charged particle travels in a field-free region $(E = 0, B = 0)$ that surrounds another region in which there is trapped magnetic flux Φ. Then, on completing a closed loop, the particle's wave function will acquire an additional phase factor. But the wave function must be single-valued at any point in space.

Thus the motion of charges in the fields free region leads to non-dissipative persistent currents. In 1959, Y. Aharonov and D. Bohm [3] theoretically predicted that a relative phase shift can exist even when the electron beams pass only through spaces free of electric field *(E)* and magnetic field *(B)*. It refers to the phenomena where the wave function of the charged (particle) quantum particle acquires a phase due to the vector potential along its path, leading to inference chances caused by the magnetic flux. If θ is the phase, then a wave function $e^{ik.r}$ can become $e^{i(k.r+\theta)}$. It should be mentioned that persistent current is a non-dissipative equilibrium property for all states below Fermi energy. In fact, persistent currents and consequence of Aharonov-Bohm [3] effect, and according to its effect, which is a quantum mechanical phenomena, an electrically charged particle is affected by an electromagnetic potential despite being confined to a region in which both the electric field *(E)* and the magnetic field *(B)* are zero.

So far, a large number of conventional and unconventional superconductors have been studied both experimentally and theoretically.

To this, a very recent addition is the superconductivity of two-dimensional atomic layers of Kagome metal. The structure of this two-dimensional Kagome crystal resembles a traditional Japanese basket weave called Kagome [4, 5, 6, 7].

In fact, Kagome metals are a promising system of materials to explore when looking for interesting phases of matter [9][10]. The quasi-two-dimensional Kagome materials such as KV_3Sb_5; RbV_3Sb_5; $CsVb_3Sb_5$, are important examples of Kagome superconductors. These present a new quantum platform to investigate the correlation between electron correlations effects, topology and geometric frustration [10]. The superconducting properties of Kagome metals, especially the important pairing symmetries and the interplay between superconductivity and the charge density wave state, have been reviewed and studied experimentally and theoretically. It is found that each Kagome metal in this family of materials has different critical parameters.

For instance, for $CsVb_3Sb_5$, the critical magnetic field $H_c = 0.4T$ (T = Tesla) which is quite small; the zero-field $(H_c = 0)$ resistivity *P(T)* variation leads to the superconducting *(SC)* ground state at $T_c = 2.3K$. The measurements showed well-defined Meissner effect and the specific heat showed sharp entropy anomaly at the superconducting transition (*SC-t*). It is found that the resistivity *P(T)* of $K V_3Sb_5$ drops to zero at $T_c = 0.93K$; and for RbV_3Sb_5, $P(T) \rightarrow 0$ at $T_c = 0.75K$. Thus, this family of materials are superconducting at quite low transition temperatures [10].

Recently, it is found that a superconductivity theory proposed by Wuzburg physics team has been validated in a universal experiment that showed that the Cooper pairs display wave-like distortions within the sublattices in Kagome metals [11]. In fact, the unique crystal geometry of Kagome metals combines in it unique distinctive properties such as electronic, magnetic, and superconducting.

In a recent experiment on Kagome metals, it is found that when the Kagome metals are in the superconducting state, an exotic metal harbours charge carriers which appear to have 4 and 6 times the charge of a single electron, suggesting the formation of Cooper pair molecules [9]. The bulk *CVS* has a transition temperature $T_c = 2.5K$, and its resistivity *P(T)* starts decreasing at about *4.0K*, and vanished at about *1K*. For a thin superconducting compound, such a drop in resistivity, spread over a temperature gap, and well above the transition temperature is very much expected. This marks a fluctuating regime of superconductivity in which Cooper pairs are present in a disorganized fashion.

It should be a general understanding that charge- 2e Cooper pairs should persist up to *4K*, and the resistance oscillation should be $\frac{h}{2e} = \phi_o$. Instead, the experimental observations showed evidence of resistance oscillations at the first period $\frac{h}{4e}$ between *1K* and *2K*; and flux period of $\frac{h}{6e}$ between *2K* and *3K*, with the $\frac{h}{2e}$ period becoming dominant in the zero-resistance regime only below *1K*.

This means in different temperature ranges, and in the superconducting state, Cooper pairs with charge 2e and combinations of Cooper pairs with charge 4e and 6e could carry the superconducting current; and this implies that the periodicities $\frac{h}{4e}$ and $\frac{h}{6e}$ confirm the possibility that two, and or three Cooper pairs, somehow coalesce into Cooper molecules, with a total charge *4e* and/or *6e.* A flux periodicity of $\frac{h}{2e}$ was experimentally observed long ago [8].

Obviously, observation of new periodicities, $\frac{h}{4e}$ and $\frac{h}{6e}$ point to the existence of some new type of exotic interaction between electrons in the Cooper pairs and Cooper molecules. The question is as to what do the Cooper pairs look like as they orbit around each other in Cooper molecules. Thus, composite quasi-particles could lead to the understanding of superconductivity in Kagome metals. We have now to decide how to write the Hamiltonian of the system in which Cooper pairs and Cooper molecules may exist simultaneously, and the superconducting state is spread over a range of temperatures such that the Cooper pairs and Cooper molecules play a dominant role over a range of temperatures. Since the Cooper pairs and molecules are composed of an even number of fermions, they can be effectively treated as bosons. The theory of second quantization and the Bogoliubov theory for interacting bosons leading to superfluidity can be used to obtain the quasi-particle energy (E_k) for such an assembly; and then obtain the expressions for specific heat (C_v) and transition temperature (T_c) [12, 13] [14] [15–18].

2. THEORETICAL DERIVATIONS

The superconducting state in Kagome metal superconductors seems to be composed of two-electron pairs and 4-electron and 6-electron Cooper molecules [7]. Recent experimental observations led to the existence of oscillations at a flux period of $\frac{h}{6e}$ between *2K* and *3K* and of $\frac{h}{4e}$ between *1K* and *2K*, the $\frac{h}{2e}$ period becoming dominant in the zero resistance regime below *1K* only. Hence, the periodicities $\frac{h}{4e}$ and $\frac{h}{6e}$ could emphatically mean that the two (2×2e) and/or three (3×2e) Cooper pairs may coalesce into Cooper molecules with Cooper molecules having a total charge of 4e and or 6e. A Cooper pair, being composed of two fermions, can behave as an effective boson, and similarly, the Cooper molecules can behave as bosons. Thus, at very low temperatures in the range *0K* to *3K;* the superconducting state can be described as the condensed state of free bosons that can flow without dissipation, leading to very large superconducting currents. Although in the different temperature ranges, the composition of the charge carriers differs, but all of them behave as effective bosons. Consequently, the theory of second quantization is used to write down the Hamiltonian *(H)* of the system of bosons, and the Bogoliubov canonical transformation is used to diagonalize the Hamiltonian *(H)* to obtain the quasi-particle energy of the system.

3. IAGONILIZATION OF THE HAMILTONIAN FOR A SYSTEM OF INTERACTING PARTICLES USING THEORY OF SECOND QUANTIZATION

The principle of second quantization are used to write down the Hamiltonian *(H)* of a system of iteracting particles. The Hamiltonian is composed of the kinetic energy and potential energy. It is assumed that the potential energy is due to pairing interaction between the particle and the three or more particle interactions are neglected assuming that they are very small when compared to the pair interaction. Such an assembly of particles is said to be weakly interacting. There are two types of particles, fermions with spin $\frac{1}{2}$, and bosons with integer spins (0, 1, 2, …). Hamiltonians for both the systems can be written and diagonalized.

For a weakly interacting assembly of bossons, the Hamiltonian is written as,

H = Kinetic energy + Potential energy

$$H = \sum_k \in_k a_k^+ a_k + \frac{1}{2}\sum G_{k_1 - k_1'} a_{k_1}^+ a_{k_2}^+ a_{k_2'} a_{k_1'} \tag{1}$$

Here, a_k^+ and a_k are the creation and annihilation operators that satisfy the usual commutation relations for bosons. In the second term of Eq.(1), momentum conservation is assumed when the sum is carried out over all values of momenta k_1, k_2, k_1', k_2' such that,

$$k_1 + k_2 = k_1' + k_2^1 \tag{2}$$

In the interaction process (second term of Eq. (1)) leading to scattering, it is clear that a particle with momentum k_1' is destroyed $\left(a_{k_1}'\right)$ and it goes to reappear as a particle with momentum $k_1\left(a_{k_1}^+\right)$ such that the momentum transfer is $k_1 - k_1'$. Similarly, the particle with momentum k_2' is destroyed $\left(a_{k_2'}\right)$ and reappears as a particle with momentum $k_2\left(a_{k_2}^+\right)$, such that the momentum transfer to this second particle is $k_2 - k_2'$. For the conservation of momentum of these two particles, the momentum transfer to both the particles must be equal in magnitude but opposite in sign, i.e.,

$$k_1 - k_1' = -\left(k_2 - k_2'\right) = -k_2 + k_2'$$

Or $\quad k_1 + k_2 = k_1' + k_2'$

The G in Eq. (1) represents the interaction between the particle with momentum k_1 and the particle with momentum k_2 and is given by Eq. (3), where $G_{k_1',k_2'}^{k_1,k_2}$ is the Fourier transform of G

$$G_{k_1',k_2'}^{k_1,k_2} = \iint \psi_{k_1}^*(r_1)\psi_{k_2}^*(r_2)G(r_1,r_2)\psi_{k_1'}(r_1)\psi_{k_2'}(r_2) \tag{3}$$

where $\psi' s$ are the wave functions associated with the particles, and $G(r_1, r_2)$ represents the form of two-body interaction between a pair of particles, and also,

$$G_{-k} = G_k \tag{4}$$

which asserts the in-variance of the two-body interaction G under time reversal. If there is no interaction between the particles, then $G = 0$; the ground state will be $k = 0$, and hence all the particles will be in the state $k = 0$, and this state is called condensed state or zero-momentum state (ZMS).

The total number of particles, N , can be distributed among the excited states with $k \neq 0$, and the condensed state $k = 0$, and hence one can write:

$$N = N_0 + \sum_{k \neq 0} a_k^+ a_k \tag{5}$$

where N_o= number of particles in the state $k = 0$.

The Hamiltonian H is diagonalized by using the Bogoliubov canonical transformation to obtain stationary states or to yield a system of non-interacting quasi-particles whose energy spectrum depends on the coefficients of the Bogoliubov canonical transformation, also called the Bogoliubov-Valatin transformation. It should be understood that the Bogoliubov-de Gennes Hamiltonian is a mean-field Hamiltonian (mean-field means that the particles move in an average potential created by the interactions between all the particles. It is treated as aconstant potential, say, V_o), i.e, a one-body quadratic Hamiltonian in which the Hamiltonian is composed of the products of creation operators (a^+) and annihilation operators (a) [15-18].

Now, to diagonalize the Hamiltonian H using the Bogoliubov canonical transformations, new operators (α^0) are introduced and they connect with the old operators (a) via the transformation constants u_k and v_k which are real constants. i.e, $u_{-k} = u_k$ and $v_{-k} = v_k$. i.e, We write:

$$\alpha_k = u_k a_k - v_k a_{-k}^+ \tag{6}$$

With the condition that ,

$$u_k^2 - v_k^2 = 1 \tag{7}$$

Here $(\alpha_k ' s)$ satisfy the same commutation laws as the (a_k), and these laws are for bosons since when two fermions (electron-electron or electron-hole) combine, they effectively behave as bosons. The Hermitian conjugate of Eq. (6) can be solved to give:

$$a_k = u_k \alpha_k + v_k \alpha_{-k}^+ \tag{8}$$

Commutation laws for boson operators are:

$$\begin{aligned}&\left[a_k, a_k^+\right] = a_k a_k^+ - a_k^+ a_k = 1\\&\left[a_k, a_k\right] = \left[a_k^+, a_k^+\right] = 0\\&\left[a_k, a_{k_2}^+\right] = \delta_{k_1 k_2}\end{aligned} \tag{9}$$

Using the values of (a_k) and (a_k^+) from Eq. (8) and the commutation laws in Eq. (9), the Hamiltonian H can be written as:

$$H = u + H_{11} + H_{20} \tag{10}$$

Where:

$$u = \frac{1}{2} N^2 G_o + \sum_{k \neq 0} \left[(\in_k + NG_k) v_k^2 + NG_k u_k v_k \right] \tag{11}$$

$$H_{11} = \sum_{k \neq 0} \left[(\in_k + NG_k)(u_k^2 + v_k^2) + 2NG_k u_k v_k \right] \alpha_k^+ \alpha_k \tag{12}$$

$$H_{20} = \sum_{k \neq 0} \left[(\in_k + NG_k) u_k v_k + \frac{1}{2} NG_k (u_k^2 + v_k^2) \right] (\alpha_k \alpha_{-k} + \alpha_k^+ \alpha_{-k}^+) \tag{13}$$

In writing the values of H, terms of higher orders in N_o have been dropped since they give corrections of higher order in G which is assumed to be small parameter.

It is evident that H will get diagonalized if we arrange things such that $H_{20} = 0$. The Hamiltonian H then becomes,

$$H = u + \sum_k E_k \alpha_k^+ \alpha_k \tag{14}$$

And this Hamiltonian describes a set of independent quasi-particles of energy E_k, such that:

$$E_k = (\in_k + NG_k)(u_k^2 + v_k^2) + 2NG_k u_k v_k \tag{15}$$

The condition for vanishing of H_{20} gives:

$$2(\in_k + NG_k) u_k v_k + NG_k (u_k^2 + v_k^2) = 0 \tag{16}$$

Eqs. (7) and (16) are to be solved simultaneously for u_k and v_k to get the dispersion formula for E_k .

We now let:

$$u_k = cosh\,x \text{ and } v_k = sinh\,x \text{, since } u_k^2 - v_k^2 = 1; \tag{17}$$

$$2u_k v_k = sinh\,2x \tag{18}$$

$$u_k^2 + v_k^2 = cosh\,2x \tag{19}$$

$$\left[cosh \quad x = \frac{e^x + e^{-x}}{2}\right] and \left[sinh \quad x = \frac{e^x - e^{-x}}{2}\right]$$

Thus from Eq. (16), we get:

$$tanh\,x = -\frac{NG_k}{\in_k + NG_k} \tag{20}$$

And from Eq. (20), we can write:

$$sinh\,2x = -\frac{NG_k}{\left[(\in_k + NG_k)^2 - N^2 G_k^2\right]^{\frac{1}{2}}} \tag{21}$$

$$cosh\,2x = -\frac{(\in_k + NG_k)}{\left[(\in_k + NG_k)^2 - N^2 G_k^2\right]^{\frac{1}{2}}} \tag{22}$$

Substituting Eqs. (21) and (22) into (18) and (19) give the values of $u_k v_k$ and $u_k^2 + v_k^2$, and substituting these in Eq. (16) gives the dispersion formula for E_k as:

$$E_k = \left[(\in_k + NG_k)^2 - N^2 G_k^2\right]^{\frac{1}{2}} \tag{23}$$

$$E_k = \left[\in_k^2 + 2N \in_k G_k\right]^{\frac{1}{2}} \tag{24}$$

Now in the limit $k \to \infty$, in Eq. (23), the second term goes to zero much faster than the first term, and hence:

$$E_k \xrightarrow[k\to\infty]{} (\in_k + NG_k) \tag{25}$$

And in the limit $k \to 0$, in Eq. (24), the first term goes to zero faster than the second term, and hence:

$$E_k \xrightarrow[k \to 0]{} \left(2NG_o \in_k\right)^{\frac{1}{2}} \tag{26}$$

It is to be understood that the non-singular two-body potential goes to zero as $k \to \infty$; i.e.,

$$Lim.k \to \infty, G_k = 0 \tag{27}$$

Now, to bring in the temperature dependence for the energy excitation formula, we multiply E_k by the many-body thermal activation factor: $E = E_k e^{-\frac{E_k}{\kappa T}}$ (here κ is the Boltzmann constant) and finally write the quasi-particle energy dispersion formula as E, that is,

$$E = E_k e^{-\frac{E_k}{\kappa T}} \tag{28}$$

The specific heat C_v is written as:

$$C_v = \left(\frac{\partial E}{\partial T}\right) \tag{29}$$

The transition temperature T_c is obtained as:

$$\left[\frac{\partial C_v}{\partial T}\right]_{T=T_c} = 0 \tag{30}$$

4. CALCULATIONS

Recent experimental observations [7] show that the superconducting state in the Kagome metals super conductors have different types of Cooper pairs in the temperature ranges $0K$ to $1.0K$, $1.0K$ to $2.0K$ and $2.0K$ to $3.0K$. In the two electron Cooper pair in the temperature range $0K$ to $1.0K$, the two-body interaction potential is denoted by G_0 and it is assumed to be constant. Similarly in the temperature range $1.0K$ to $2.0K$ in which there exists Cooper molecules of charge $4e$. The interaction potential between two Cooper pairs that constitute the Cooper molecule is denoted by $G_1 = 2G_0$. In the temperature range $2.0K$ to $3.0K$, the Cooper molecule will be of charge $6e$ and the interaction potential will be denoted by $G_2 = 3G_0$ which is a constant.

Now in the range $0K$ to $1.0K$, E_k will be given by Eq. (26), i.e,

$$E_k^{(o)} = \left(2NG_o \times \in_k\right)^{\frac{1}{2}} \tag{31}$$

In the temperature range $1.0K$ to $2.0K$ we can write wring e.g (24),

$$E_k^{(1)} = \left(\in_k^2 + 2N \in_k \times 2G_o\right)^{\frac{1}{2}} \tag{32}$$

And the temperature range $2.0K$ to $3.0K$, we get,

$$E_k^{(2)} = \left(\in_k^2 + 2N \in_k \times 3G_o\right)^{\frac{1}{2}} \tag{33}$$

Using many-body thermal activation factor, the quasi particles energy dispersion formula can be written as;

$$E_o = E_k^{(o)} e^{-\frac{E_k^{(o)}}{\kappa T}} \quad \text{(Where is } \kappa \text{ Boltzmann constant)} \tag{34}$$

$$E_o = \left(2NG_o \in_k\right)^{\frac{1}{2}} e^{-\frac{E_k^{(o)}}{\kappa T}} \tag{35}$$

$$E_o = E_k^{(1)} e^{-\frac{E_k^{(1)}}{\kappa T}} = \left(\in_k^1 + 2N \in_k \times 2G_o\right)^{\frac{1}{2}} e^{-\frac{E_k^{(1)}}{\kappa T}} \tag{36}$$

$$E_o = E_k^{(2)} e^{-\frac{E_k^{(2)}}{\kappa T}} = \left(\in_k^2 + 2N \in_k \times 3G_o\right)^{\frac{1}{2}} e^{-\frac{E_k^{(2)}}{\kappa T}} \tag{37}$$

As a sample calculation, first Eq. (35) can be used to calculate specific heat (C_v), and transition temperature (T_c). For doing the calculations, we use the following values for different parameters:

- Cooper pair interaction $G_o = 10^{-3} eV$ for metals.
- N = Number of cooper pairs $\cong 10^6$
- $\in_k$ = Energy level of cooper pairs $\cong (10-70)meV = (10-70) \times 10^{-3} eV$. Since we are dealing with a very low temperature superconducting state, the value of $\in_k$ is chosen to be $10\text{x}10^{-3} eV = 10^{-2} eV$.

The specific heat (C_v) is calculated by using equation (35) as;

$$C_v = \frac{\partial E_o}{\partial T} = \left(E_k^o\right)^2 \frac{1}{\kappa T^2} e^{-\frac{E_k^o}{\kappa T}} \tag{38}$$

The transition temperature is calculated by writing,

$$\left(\frac{\partial C_v}{\partial T}\right)_{T=T_c} = 0$$

Which leads to T_c as,

$$T_c = \frac{E_k^o}{2\kappa} \tag{39}$$

Now the binding energy of a Cooper pair is the energy of the gap Δ that exists in the super- conducting state. For Al, $\Delta \cong 3.4 \times 10^{-4} eV$. Thus, for Kagome metals, we can assume that $G_o \cong 10^{-4} eV$. The magnitude of the Quasi particle energy $(\in_k)$ is obtained by the Quasi particle map [20], and it turns out to be, $\in_k = 40 \mu eV = 40 \times 10^{-6} eV = 4 \times 10^{-5} eV = 4 \times 10^{-5} \times 1.6 \times 10^{-12} erg$. Thus we get;

$$E_k^o = (2NG_o \in_k)^{\frac{1}{2}} \cong 14.31 \times 10^{-14} erg \tag{40}$$

Hence, to calculate the values of E_k^o for the involved parameters *N*, G_O and $\in_k$; these parameters are as follows:

- Interaction energy of the Cooper pair, $G_0 \cong 10^{-4} eV$.
- Number of Cooper pairs *(N)*. In general, there may be 10^6 *to* 10^7 electrons in between a Cooper pair such that electrons may form Cooper pairs with other electrons. There is no reason to believe that all these electrons may form Cooper pairs. Generally the number of electrons that form Cooper pairs are 10^{-4} fraction [2]. Thus, *N* may vary between 10^2 *to* 10^3
- Quasi particle energy $\in_k = 40 \mu eV = 40 \times 10^{-6} eV$. It could as well be $4 \times 10^{-6} eV$[19,20]. Using these parameters, we have calculated the values of E_k^o for different values of G_0 i.e;

Table 1. Showing variation of E_k^o with G_o

$G_o(eV) \times 10^{-4}$	$E_k^o(erg) \times 10^{-15}$
0.1	0.451
0.2	0.638
0.25	0.714
0.33	0.824
1	1.428

Now these values of E_k^o can be used to calculate C_V from Eq. (38), and T_C from Eq. (39). A sample calculation for C_v for $E_k^o = 1.428 \times 10^{-15} erg, and\ T = 1.0K$ gives $C_v = 4.623 \times 10^{-26} J/K$. Values of C_v can be calculated for using different values of G_o, E_k^o in the temperature range T=1.0K to 3.0K.

Of greater importance are the values of T_C for different values of G_O and these we got from Eq. (39).

Table 2. Showing variation of T_C with G_O

$G_o(eV) \times 10^{-4}$	T_C (K)
0.1	1.636
0.2	2.31
0.25	2.987
0.33	2.987
1	5.174

5. RESULTS AND CONCLUSIONS

Table 1 shows that Quasi particle energy increases as the interaction energy G_O increases in the Cooper pairs and it should be so since stronger interaction leads to more energy in the system.

The magnitude of specific heat turns out to be very small as it should be in the superconducting state. In the same view point the entropy will also be very small since these values must be very low at very low transition temperatures.

Table 2 shows the variation of T_C with G_O. It indicates that the values of T_C increase as G_O increases. Smaller G_O value means weak interaction in the Cooper pairs and thus, to break the Cooper pair we need less energy which means smaller T_C value. Whereas for strong interaction (larger G_O value), we need more energy to break the pair and hence T_C is large comparatively. These results are in line with the T_C values that one can obtain from the general expression for T_C which is $T_c \cong \frac{2\Delta(0)}{3.5\kappa}$ [1,2].

From **Table 2** it is clear that for T_C in range $0K\ to\ 1.0K$ when the flux quantum periodicity is $\frac{h}{2e}$,the values of G_O may lie between $10^{-5}eV\ and\ 10^{-6}eV$. In the range for T_C between $1.0K$ and $2.0K$, when the flux quantum periodicity is $\frac{h}{4e}$, the value of G_O may be between $10^{-5}eV\ \ and\ \ \ 0.2\times10^{-4}$eV. In the range for T_C between $2.0K$ and $3.0K$ when the flux quantum periodicity is $\frac{h}{6e}$ the value of G_O may be between $0.2\times10^{-4}eV\ to\ 10^{-4}eV$. This is what has been observed in Kagome metals superconductors. Our calculations lead to the possible strength of interaction that may exist in the copper molecules, and this calls for experimental confirmation.

Due to increase in the number of electrons in the Cooper molecules, we can try to study the effect of increased Coulomb interaction between the electrons on G_O and T_C in future studies [21]

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