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Prediction of Monkeypox Cases Using the Adam Bashforth–Moulton Numerical Approach to the Verhulst Logistics Model

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ABSTRACT

Monkeypox is a viral disease that resembles smallpox in general, usually called monkey pox. Monkeypox (MPXV) is a zoonotic orthopoxvirus (OPX) that is endemic to West and Central Africa. Transmission of monkey pox (monkeypox) to humans can occur through direct contact between humans and infected animals or by eating meat that is not cooked properly. This research discusses the Logistic Equation using the Adam Bashforth Moulton method to predict monkey pox cases, which is then solved first using the 4th order Runge Kutta method to obtain an initial solution, then using Adam Bashforth as a predictor and Adam Moulton as a corrector. Logistic equation for population growth with carrying capacity in Africa K= 116,064,589,944 with step size h = 1, with a growth rate of 1%. The numerical solution to the logistic equation with the growth of monkey pox cases at time t=24 with a step size h=1 is 12,948 people.

Keywords: Logistics Equation, Runge Kutta Method, Adam Bashforth Moulton Method.

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1. INTRODUCTION

Monkeypox is a viral disease that resembles smallpox in general. Monkeypox (MPXV) is a zoonotic orthopoxvirus (OPX) that is endemic to West and Central Africa. Monkeypox virus was discovered in 1958, when it was isolated from lesions of vesiculo-pustular disease commonly found in monkeys at the State Serum Institute, Copenhagen. Close similarities between smallpox or variola and monkeypox in primates [1]. Before 1970, monkeypox, a disease caused by the Orthopoxvirus, monkeypox virus (MPXV), was only known in nonhuman hosts. As of 21 May 2022, there were 92 laboratory-confirmed cases and 28 suspected monkeypox cases still under investigation that had been reported by WHO from 12 member countries that were non-endemic to the monkeypox virus. In West Africa, the death rate due to monkeypox is 3.6% and in the Cango Basin, it is 10.5%. Data reported by WHO in endemic countries between 13 May and 21 May 2022 include Cameroon with 25 cumulative cases, Central Africa with 6 cumulative cases, Democratic Republic of the Congo with 1,238 cumulative cases and 57 cumulative deaths, and finally Nigeria with 46 cumulative cases. Based on the description of the data above, predictions are needed regarding the estimated number of cases to enable planning appropriate actions to overcome the increase in this virus. There are many events for which differential equations can be created, such as spring models, electric circuit models, disease spread models, population growth models and others. There are four types of population growth models, namely discrete models, exponential models, population growth models with age distribution, and logistic models. Of the four types of models, the logistic model introduced by Verhulst is the most accurate population growth model. The logistic population growth model according to Verhulst is a mathematical model with nonlinear differential equations [2].

In some forms of differential equations, analytical solutions are not available. On the other hand, numerical methods can be an alternative solution when analytical methods are not available. Related research conducted by Zhi Pei [3] states that multi-step methods include various types of methods, such as the Adams method, Milne method, and Hamming method. This research focuses on the application of the Adams method to solve numerical problems in population growth prediction. In this case, the Adams method used is Adams Bashforth as predictor and Adams Moulton as corrector, which functions to solve mathematical equations. Furthermore, similar research was carried out by Kumala Sari [4] The Adams Bashforth-Moulton method was carried out by first determining the predicted value using the Adams Bashforth method, then the value was corrected using the Adams Moulton method. This process involves comparing relative errors with stopping criteria. The iteration will continue until it reaches the specified interval, as long as the relative error is still greater than the stopping criterion.

Based on the description above, in general, there are two types of numerical methods for solving differential equations, namely one-step methods and many-step methods. The Adams method is an example of a multi-step method, which is divided into two types, namely the open method or Adams-Bashforth method and the closed method or Adams-Moulton method [5]. One-step methods require an initial value, while multi-step methods require multiple initial values. The multi-step method is often also called the prediction correction method because the process of obtaining a solution has two stages, namely the first stage uses the prediction equation to obtain the predicted value, and the second stage uses the prediction equation to obtain the predicted value. get the corrected value. Correction equation.

The frequently used multi-step method is the Adams-Bashforth-Moulton method [6]. The Adams-Bashforth method can be used as a predictor equation, and the Adams-Moulton method can be used as a corrector equation in solving a differential equation. This research will discuss the application of the Adams Bashforth-Moulton method to predict monkey pox cases using the Verhults logistic equation.

2. MATERIALS AND METHODS

2.1. Verhulst Logistic Model

The logistic growth model is an approach used to describe the dynamics of population growth by considering the limited resources available in the environment. In contrast to the exponential growth model which assumes that the population can grow indefinitely, the logistic model is more realistic because it takes into account environmental factors that can limit growth. In this context, the intrinsic growth rate (m) is an important parameter that reflects the ability of a population to develop. This rate is influenced by a variety of factors, including food availability, living space, and mortality rates, all of which contribute to population dynamics [7].

In the logistic model, the intrinsic growth rate is used to determine the maximum capacity of the population, known as the environmental limit capacity (K) [8]. This capacity serves as a constraint on population growth as the population approaches the value of K. Thus, the logistic model not only provides an idea of how the population could grow under ideal conditions, but also how that growth will slow as resources become more limited. The mathematical equation underlying the logistic model can be written as follows:

$$(1) \qquad \frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P$$

Here is the analytical solution:

$$(2) \qquad P(t) = \frac{K}{1 + Ae^{-mt}}$$

With:

K : Carrying Capacity
$P(t)$: Population in year t
A : Initial value
m : Population growth rate
t : Time period

2.2. Runge-Kutta Method

A relatively simple yet fairly accurate method that is often used is the Runge-Kutta method method that is often used is the Runge-Kutta method. The Runge-Kutta method is an alternative to the Taylor series method which does not require many derivative calculations. This approach is designed to achieve a higher level of precision without having to calculate many derivatives, but rather by evaluating functions $f(x, y)$ directly, at selected points in each step interval [9]. The fourth level Runge-Kutta method is also a well-known method and is often used in practice. The fourth order Runge-Kutta method is also used as a preliminary to obtain the initial values needed later in the fourth order Adams-Bashforth-Moulton method. The following is the equation of the fourth order Runge-Kutta method:

(3)

$$k_1 = hf(x_r, y_r)$$
$$k_2 = hf\left(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1\right)$$
$$k_3 = hf\left(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2\right)$$
$$k_4 = hf(x_r + h, y_r + k_3)$$
$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

2.3. Adams-Bashforth-Moulton Method

Differential numerization methods are widely used in the field of science and technology to solve differential equations. The two main types of methods are one-step methods and multi-step methods. The one-step method only uses data from one previous point to calculate the next solution, as in Euler's method. In contrast, multi-step methods utilize data from several previous points, so they generally reach convergence more quickly with the same level of accuracy [10]. One example of a popular multi-step method is the Adams-Bashforth four-step method, which uses interpolation polynomials to estimate the value of a function at intertime points. The Adams-Bashforth-Moulton method also falls into this category, where it makes predictions and then corrects them to increase accuracy. To obtain the initial solutions required by multi-step methods, simple one-step methods such as Euler, Heun, Taylor series, or Runge-Kutta are often used, all of which play an important role in producing realistic and accurate simulations in a variety of practical applications [11].

The multi-step method is often called the predictor-corrector method, because the solution process uses predictor and corrector equations without requiring the calculation of function derivatives [12]. The predictor equation, or first equation, is usually used to estimate the initial solution, obtaining a first approximation for y_{r+1} while the corrector equation or second equation is usually used to obtain the corrected value (second approximation for y_{r+1}). The most famous multi-step method is the fourth order Adams-Bashforth-Moulton method.

The truncation error of the fourth order Adams-Bashforth-Moulton method is smaller than the truncation error of the second order and third order Adams-Bashforth-Moulton methods so it can provide a fairly accurate solution. In this method y_{r+1} obtained from y_{r+3}, y_{r+2} y_{r+1} dan y_r with the values of the 4 initial data (x_0, y_0), (x_1, y_1), $(x_2, y_2), (x_3, y_3)$ must already exist and can be calculated using the one-step method, namely the fourth order Runge-Kutta method for calculating (x_r, y_r) [13].
The predictor equation of the fourth order Adams-Bashforth method is

$$(4) \quad y^*_{r+1} = y_r + \frac{h}{24}(-9f_{r-3} + 37f_{r-2} - 59f_{r-1} + 55f_r$$

The corrective equation of the Fourth-Order Adams-Moulton Method is:

$$(5) \quad y_{r+1} = y_r + \frac{h}{24}(f_{r-2} - 5f_{r-1} + 19f_r + 9f^*_{r+1}$$

with $f^*_{r+1} = f(x_{r+1}, y^*_{r+1})$ [14]

Table 1. Monkeypox Accumulation.

No	Month	Number of Cases
1	January	1391
2	February	1460
3	March	1516
4	April	1633
5	May	1740
6	June	1842
7	July	1947
8	August	2000
9	September	2038
10	October	2182
11	November	2320
12	December	2404
13	January	2622
14	February	2868
15	March	3171
16	April	3471
17	Mei	4189
18	June	5376
19	July	6394
20	August	9453
21	September	10152

3. RESULT AND DISCUSSION

Based on the data and images below, the steps that need to be followed to obtain the predicted number of Monkeypox infections in Africa using the Adams-Bashforth-Moulton method are as follows:

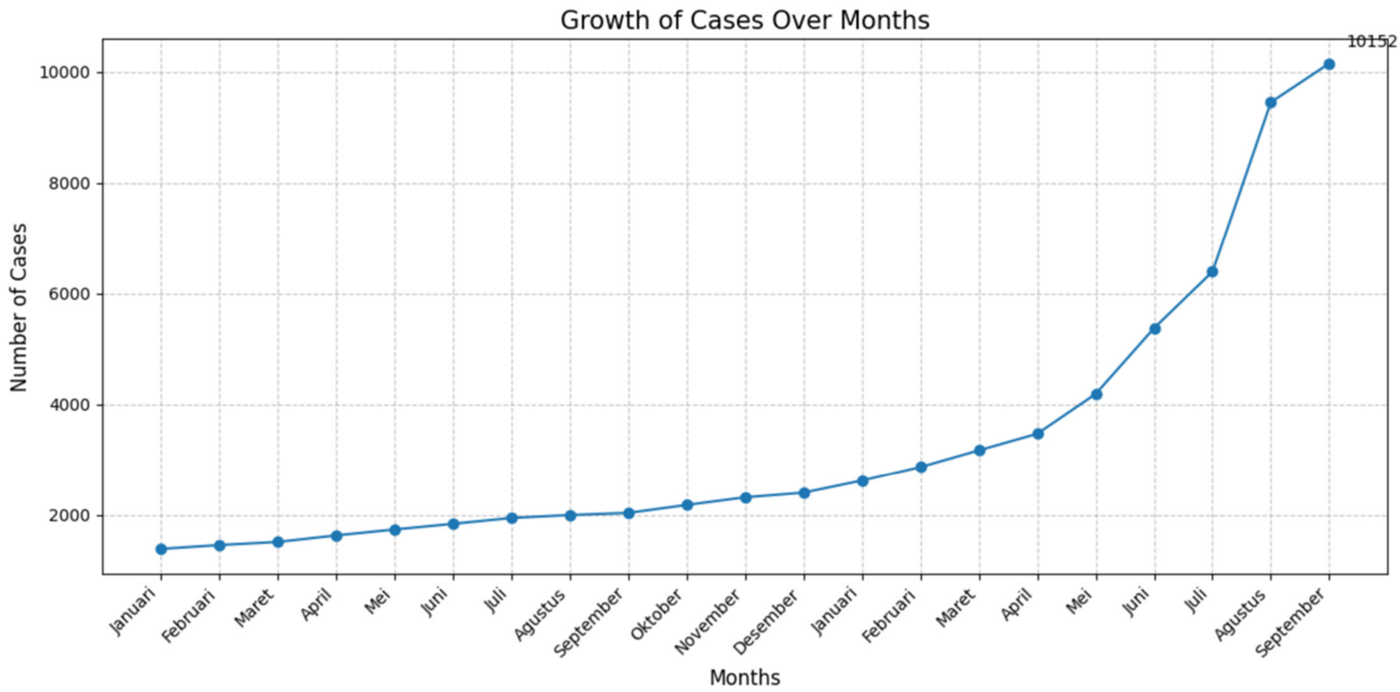

Figure 1. Monkeypox Case Growth Graph in Africa for 2024.

This graph shows the growth in the number of cases over a 21 month period, starting from January to September 2024 . The data points show the number of cases recorded each month, with a clear upward trend visible throughout the time period. From January to August, the growth in the number of cases was relatively stable, with a gradual increase each month. However, a more significant spike in cases can be seen starting in September, where the number of cases increased rapidly, reaching more than 10,000 by the end of the period. Then the data to be analyzed is below.

3.1. Data Analysis

3.1.1. Determination of Growth Rate

The determination of the growth rate is done using the analytical solution of the logistic equation. The calculation results can be seen in Table 2. The value (growth rate) mm is determined using the following equation:

$$m = \frac{1}{t} in \left(\frac{P}{P_0}\right)$$

$$m = \frac{1}{1} in \left(\frac{1460}{1391}\right) \approx 0{,}143$$

Next, the determination of the value of K (*carrying capacity*) is obtained through trial and error by substituting the estimated value of K into the Verhulst model. Since the population in Africa from 2022 to 2024 is approximately 116,064,589,944, it is assumed that $K = 116{,}064{,}589{,}944$.

Table 2. The Calculation Results.

	$P(0)$	M
Result	1391	0,143

Based on Table 2, the carrying capacity of Africa is K = 116064589944 The growth rate obtained is m = 0,143 and P(0) = 1391 as the initial value, on the interval [0, 21], with the number of steps n = 21.

$$h = \frac{b-a}{n} = \frac{21-0}{21} = 1$$

with a step size of h =1

The obtained values, such as the growth rate and carrying capacity of Africa, are substituted into the logistic equation, resulting in the following equation:

$$\frac{dP}{dt} = m\left(1 - \frac{P}{K}\right)P = 0{,}143\left(1 - \frac{P}{116064589944}\right)P$$

After obtaining the logistic equation, the next step is to find the initial solution values using the Fourth-Order Runge-Kutta method.

3.1.2. Determination of the initial solution P_0, P_1, P_2, P_3 Using the Fourth-Order Runge-Kutta Method

After determining the logistic equation, the next step is to determine the initial solution values, which are P_0, P_1, P_2 dan P_3 using the fourth-order Runge-Kutta method. With the logistic equation, initial values, and the interval already known, it is given that...

$$\frac{dP}{dt} = 0{,}143\left(1 - \frac{P}{116064589944}\right)P$$

Calculating the initial solution $P_1 P_2$ dan P_3. With the initial value $P_0 = 1391$ on the interval [0, 21] with a step size of (h = 1).

After the values K_1, K_2, K_3 dan K_4 are obtained, then these values are substituted into the fourth-order Runge-Kutta equation.

$$P_{n+1} = P_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Table 3. Initial Solution Using the Runge-Kutta Method.

n	P_n	$\frac{dP}{dt} = 0,143\left(1 - \frac{P}{116064589944}\right)P$
1	1391	**198.913**
2	1605	**213.135**
3	1852	**214.152**
4	2136	**229.536**

3.1.3. Determination of the Numerical Solution Using the Adams-Bashforth Method

After obtaining the initial solution values, the next step is to calculate the values of f_r, f_{r-1}, f_{r-2} and f_{r-3} for r = 3,4, ... , n. These values are then substituted into the Adams-Bashforth equation with a step size of $h = 1$
for $r = 3$ dan P_3 =2136

Calculate the value f_4^* and then compute the numerical solution using the Adams-Moulton method for correction.

Table 4. Numerical Solution of the Adams-Bashforth-Moulton Method in the Verhulst Logistic Equation for Monkeypox Infection Growth 2024

n	P_n	P_r	error (ε)
0		1391	
1		1460	
2		1516	1.0
3	1633,01	1633,	1.0
4	1740,02	1740	1.0
5	1842,01	1842	0.9802036028840916
6	1947,19	1947	0.9849062286902446
7	2000	2000	0.9865823281724823
8	2038,01	2038	0.9872257983915883
9	2182,34	2182	0.98851964414214
10	2320,13	2320	0.9896247530048611
11	2404,01	2404	0.990373889195969
12	2622,01	2622	0.9915147622749436
13	2868,99	2868	0.9925420396955281
14	3171,01	3171	0.9935150615930887
15	3471,30	3471	0.9943042525400335
16	4189,00	4189	0.9954626928205476
17	5376,01	5376	0.9966009897358357
18	6394,02	6394	0.99725247090555
19	9453,50	9453	0.9982133130302009
20	10152,14	10152	0.998400553511072
21	10851,18	10851	0.998400553511072
22	11550,07	11550	0.998580838454277
23	12249,01	12249	0.9986949418126841
24	12948,08	12948	0.9987975980668562

The determination of the numerical solution using the Adams-Bashforth method is substituted into the Adams-Moulton equation, and the error is calculated. The results indicate that each month, October, November, and December, shows an increase.

The next predictor value is refined using the corrector equation as follows:

$$y_{r+1} = y_r + \frac{h}{24}(f_{r-2} - 5f_{r-1} + 19f_r + 9f_{r+1}^*)$$

$$y_3^{(1)} = y_3 + \frac{h}{24}(f_{r2} - 5f_{r3} + 19f_r + 9f_{r+1}^*)$$
$$= 1990.7$$

The predictor and corrector values are then used to calculate the relative error, resulting in:

$$\frac{\left|y_3^{(1)} - y_3^{(0)}\right|}{\left|y_3^{(1)}\right|} = \frac{|1990.7 - 1633{,}01|}{|1990.7|} = 0.179700304940$$

Based on the results above, it can be concluded that the relative error is smaller than the stopping criterion, so the iteration is continued until the 25 iteration as shown in Table 4. Next, an estimation solution comparison is made using the Runge-Kutta method.

Table 5. Numerical Solution of the Verhulst Logistic Equation for Monkeypox Infection Growth in 2024 Using Runge-Kutta

n	P_n	P_r	error (ε)
0	1391.00	1391	
1	1659.65	1460	
2	1971.38	1516	0.000000
3	2329.63	1633,	0.136748
4	2736.83	1740	0.300382
5	3193.90	1842	0.426595
6	3699.78	1947	0.572890
7	4251.06	2000	0.733930
8	4841.72	2038	0.900249
9	5463.20	2182	1.125531
10	6104.75	2320	1.375720
11	6754.14	2404	1.503756
12	7398.48	2622	1.631359
13	8025.30	2868	1.809541
14	8623.48	3171	1.821692
15	9183.93	3471	1.798223
16	9700.07	4189	1.719483

17	10167.94	5376	1.645903
18	10585.97	6394	1.315606
19	10954.71	9453	0.891357
20	11276.30	10152	0.655610
21	11554.00	10851	0.158861
22	11791.75	11550	0.110747
23	11993.81	12249	0.064786
24	12164.48	12948	0.020931

Based on the displayed data, the Runge-Kutta method used to estimate population growth shows a difference between the estimated value (Pn) and the actual value (Pr), with the initial value starting at 1391. At step 1, the difference begins to appear with Pn =1659.65 and Pr =1460. The difference becomes more pronounced in the middle of the simulation, especially at steps 8-14, where Pn increases more rapidly than Pr (e.g., at step 10: Pn =6104.75, Pr =2320). However, the gap starts to narrow towards the end of the simulation (steps 20-24) with final values of Pn =12164.48 vs Pr =12948, indicating that the model overestimates at the beginning and underestimates at the end. In conclusion, the Adams-Bashforth-Moulton method provides more accurate results in this case, as it produces smaller errors and predicted values that are closer to the actual values compared to the Runge-Kutta method. Next, the comparison graph can be seen in the image below.

Th

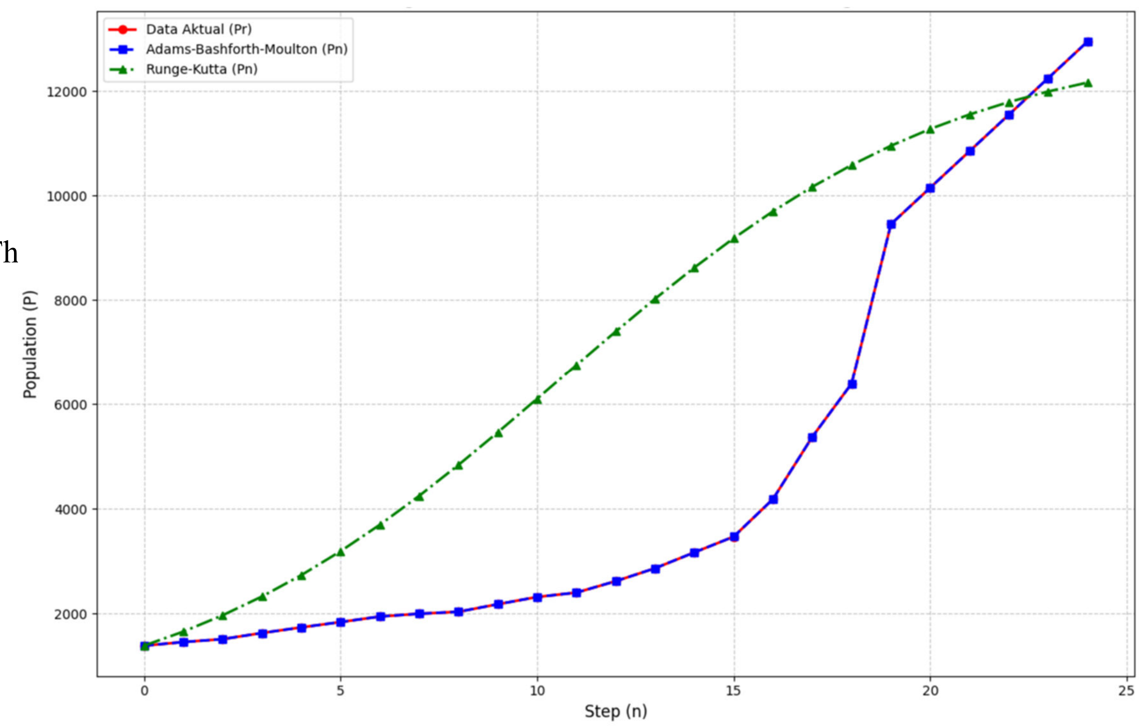

Figure 2. Comparison graph of the Adams-Bashforth-Moulton and Runge-Kutta methods.

Based on the comparison graph of the Adams-Bashforth-Moulton and Runge-Kutta methods, it can be seen that the Adams-Bashforth-Moulton method (blue line) provides very accurate results as it closely aligns with the actual data (red line). In contrast, the Runge-Kutta method (green line) shows significant deviation, with values being much higher than the actual data (overestimating) in steps 0-15, and lower than the actual data (underestimating) in steps 15-24. The largest difference is observed in steps 10-15, where the green line reaches a population of around 6000-8000, while the actual data remains in the range of 2000-3000. This indicates that, for this population growth case, the Adams-Bashforth-Moulton method provides far better estimates compared to the Runge-Kutta method.

4. CONCLUSIONS

The conclusion drawn from this study is that the Adams-Bashforth-Moulton Method and the Runge-Kutta Method with the Verhulst Model in predicting (estimating) the growth of Monkeypox cases show an increase in the number of cases at each step. Furthermore, based on the comparison between the Adams-Bashforth-Moulton Method and the Runge-Kutta Method, it can be concluded that the Adams-Bashforth-Moulton Method is more efficient compared to the Runge-Kutta Method and performs better in solving the Verhulst Model differential equation because it is more accurate. This is evident from the graph, where the Adams-Bashforth-Moulton method closely aligns with the actual data, whereas the Runge-Kutta method shows significant deviation with overestimating in the early steps and underestimating in the later steps.

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