Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 185 (2023) 83-104 EISSN 2392-2192

Numerical Investigations of the Impact of Peclet Number on the Transient Nonlinear Thermal Response of Radiating-Convecting Internally-Heated Porous Moving Fin using Method of Lines

Gbeminiyi Sobamowo[1,*], **Rotimi O. Fawumi**[2], **Ridwan O. Olagbadamosi**[3] and **Bukola O. Falomo**[4]

[1]Department of Mechanical Engineering, University of Lagos, Nigeria

[2]Department of Advanced Engineering and Management, Sheffield Hallam University, United Kingdom

[3]Department of Mechanical Engineering, Lagos State University, Epe Campus, Lagos, Nigeria

[4] Department of Mathematics, University of Lagos, Nigeria

*E-mail address:

ABSTRACT

In the present study, numerical method of lines is applied to study the impact of Peclet number on the transient nonlinear thermal behaviour of moving porous fins. Through energy analysis of the passive device, the transient thermal model of the fin is developed. The nonlinear partial differential equation is nondimensionalized to directly establish the Peclet number in the adimensional thermal model. Thereafter, the model is solved by numerical method of lines and the influences of Peclet number on the thermal response of the fins is explored. The results of the numerical investigations illustrate that the temperature of the fin is enhanced when the Peclet number is augmented. This shows that low value of Peclet number favours cooling enhancement. Also, under varying Peclet number, the extended surface thermal distribution decreases as porous, conductive-convective, conductive-radiative and porous terms increase. However, the temperature of the passive device rises as internal heat generation, ambient and surface temperatures is heightened. This study has given better physical insights and understanding of the thermal problems in extended surfaces.

(Received 07 September 2023; Accepted 04 October 2023; Date of Publication 06 October 2023)

Keywords**:** Moving fin, Peclet number, Porous Fin, Thermal analysis, Transient response, Numerical method of lines

1. INTRODUCTION

The applications of fins and spines are for augmentations of the rate of heat transfer in thermal and electronic components. The theoretical investigations of fin behaviour from to the nonlinear thermal models have been analyzed in different works. In such studies, Torabi et al. [1] Aziz and Khani [2] presented analytical solution for continuously moving convecting-radiating fin with variable thermal conductivity using differential transformation and homotopy analysis methods while Aziz and Lopez [3] performed such thermal analysis for sheet/rod.

Singh et al. [4] applied wavelet collocation solution for the same problem. Aziz and Torabi [5] studied the passive device with simultaneous variation of thermal properties. Ma et al. [6] adopted spectral collocation method for radiating–conducting porous fin with variable thermal properties.

Sun et al. [7] utilized the same method for the thermal analysis of a moving rod with variable thermal conductivity.

Kanth and Kumar [8, 9] used Haar wavelet method for the moving fin with variable thermal conductivity. Adomian decomposition method was made use by Singla and Ranjan [10]. Moradi and Rafiee [11] as well as Dogonchi and Ganji [12] explored differential transformation method (DTM) to generate for the thermal behaviour of continuously moving convecting-radiating fin with variable thermal conductivity. Sun and Ma [13] implemented collocation spectral method for moving convecting-radiating fin with variable thermal properties and internal heat generation.

In their studies, Sobamowo et al. [14] compared differential transformation and finite difference methods for the thermal problems while Ndlovu and Moitsheki [15-17] used DTM to study transient thermal problem through a moving fin with variable thermal properties. Other researchers [18-37] adopted various analytical, numerical, semi-analytical, semi-numerical and approximate analytical methods to study heat transfer in continuously moving device. However, a study on the exploration of Peclet number effects on the on the thermal responses of fins is limited in literature.

Therefore, in this work, numerical method of lines is adopted to solve the transient thermal model of a convective-radiative rectangular moving porous fin with temperature dependent thermal conductivity and internal heat generation. The numerical solutions are used to established the effect of Peclet number on the thermal behaviour of the fin.

2. MODEL DEVELOPMENT FOR THE TRANSIENT THERMAL FLOW PROCESS

Consider a porous moving fin which is internally heated. Given that the fin has length L, thickness δ and perfectly and thermally attached to a prime surface at temperature $T_{b.}$

The various assumptions for the present study can be found in our previous works [14, 29].

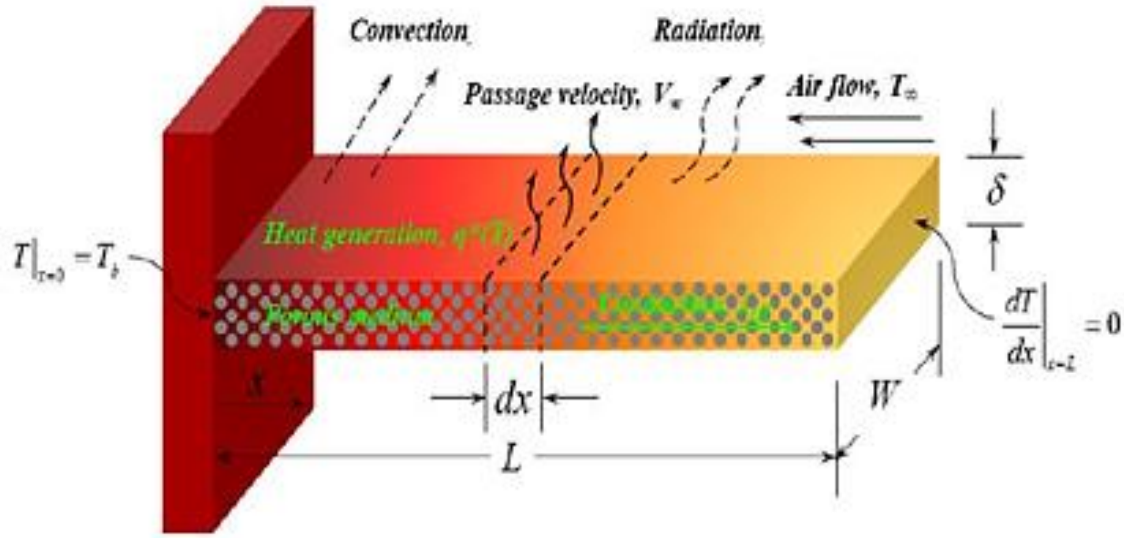

Fig. 1. Schematic of a longitudinal moving porous fin with perfect thermal contact and insulated tip

Adopting the above model assumptions, the energy analysis from Fig. (1) provides the differential equation governing the thermal model as

$$\frac{\partial}{\partial x}\left(k_{eff}\frac{\partial T}{\partial x}\right)+\frac{4\sigma\phi}{3\beta_R}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right)-\frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta}-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)$$
$$-\frac{\sigma\varepsilon P}{A_{cr}}(T^4-T_s^4)+(1-\phi)q'''(T)=\left(\rho c_p\right)_{eff}u\frac{\partial T}{\partial x}+\left(\rho c_p\right)_{eff}\frac{\partial T}{\partial t} \quad .. (1$$

The variable thermal conductivity and internal heat generation are respectively given by the linear expressions as

$$k_{eff}=(1-\phi)k_s+\phi k_f \tag{2}$$

where,

$$q'''(T)=q_o'''[1+\gamma(T-T_a)] \tag{3}$$

and

$$\left(\rho c_p\right)_{eff}=(1-\phi)\left(\rho c_p\right)_s+\phi\left(\rho c_p\right)_f \tag{4}$$

Then, the governing equation becomes

$$k_{eff}\frac{\partial^2 T}{\partial x^2}+\frac{4\sigma\phi}{3\beta_R}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right)-\frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta}-\frac{h_b P(1-\phi)(T-T_a)}{A_{cr}}$$
$$-\frac{\sigma\varepsilon_b P}{A_{cr}}\left(T^4-T_s^4\right)+(1-\phi)q_o'''[1+\gamma(T-T_a)]=\left(\rho c_p\right)_{eff}\left[u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}\right] \tag{5}$$

Linearizing the radiative term based on Roseland's approximation, we expanded T^4 about T_a with the aid of Taylor series,

$$T^4 \cong T_a^4+4T_a^3\left(T-T_a\right)+6T_a^2\left(T-T_a\right)^2+4T_a\left(T-T_a\right)^3+\ldots \tag{6}$$

If the higher order components in Eq. (6) are ignored, we have

$$T^4 \cong 4T_a^3 T-3T_a^4 \tag{7}$$

Therefore,

$$T^4-T_a^4 \cong 4T_a^3\left(T-T_a\right) \tag{8}$$

Putting Eq. (8) into Eq. (5),

$$-\frac{4\sigma\phi}{3\beta_R}\frac{\partial T^4}{\partial x}=-\frac{4\sigma\phi}{3\beta_R}\frac{\partial(4T_a^3T-3T_a^3)}{\partial x}=-\frac{16\sigma\phi T_a^3}{3\beta_R}\frac{\partial T}{\partial x} \tag{9}$$

Putting Eqs. (8) and (9) in Eq. (5),

$$k_{eff}\frac{\partial^2 T}{\partial x^2}+\frac{16\sigma\phi T_a^3}{3\beta_R}\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}}-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)$$
$$-\frac{4\sigma\varepsilon PT_a^3}{A_{cr}}(T-T_a)+(1-\phi)q_o'''[1+\gamma(T-T_a)]=\left(\rho c_p\right)_{eff}u\frac{\partial T}{\partial x}+\left(\rho c_p\right)_{eff}\frac{\partial T}{\partial t} \tag{10}$$

The initial condition is

$$T=T_0, \quad \text{when} \quad t=0, \quad \text{for} \quad 0<x<L, \tag{11}$$

The boundary conditions

$$T=T_b, \quad \text{at} \quad x=0, \text{for} \quad t>0, \tag{12}$$

$$\frac{dT}{dx}=0\text{, at} \quad x=L, \text{for} \quad t>0, \tag{13}$$

Eq. (10) can be expressed as

$$\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}}-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)$$
$$-\frac{4\sigma\varepsilon P T_a^3}{A_{cr}}(T-T_a)+(1-\phi)q_o'''[1+\gamma(T-T_a)]=(\rho c_p)_{eff}u_o\frac{\partial T}{\partial x}+(\rho c_p)_{eff}\frac{\partial T}{\partial t} \tag{14}$$

The variable internal heat generation is stated as follows

$$q'''(T)=(1-\phi)q_o'''+(1-\phi)q_o'''\gamma T-(1-\phi)q_o'''\gamma T_a \tag{15}$$

Therefore

$$\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}}-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)-\frac{4\sigma\varepsilon P T_a^3}{A_{cr}}(T-T_a)$$
$$+(1-\phi)q_o'''[1+\gamma(T-T_a)]=(\rho c_p)_{eff}u\frac{\partial T}{\partial x}+(\rho c_p)_{eff}\frac{\partial T}{\partial t} \tag{16}$$

when the like terms are collected, we have

$$\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}}-\left[\begin{array}{l}\frac{hP(1-\phi)}{A_{cr}}+\frac{4\sigma\varepsilon P T_a^3}{A_{cr}}\\-\left(\gamma(1-\phi)q_o'''\right)\end{array}\right](T-T_a)$$
$$+(1-\phi)q_o'''=(\rho c_p)_{eff}u\frac{\partial T}{\partial x}+(\rho c_p)_{eff}\frac{\partial T}{\partial t} \tag{17}$$

Which can be written as

$$\left(1+\frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}k_{eff}}-\left[\begin{array}{l}\frac{hP(1-\phi)}{A_{cr}k_{eff}}+\frac{4\sigma\varepsilon P T_a^3}{A_{cr}k_{eff}}\\-\frac{\left(\gamma(1-\phi)q_o'''\right)}{k_{eff}}\end{array}\right](T-T_a)$$
$$+\frac{(1-\phi)q_o'''}{k_{eff}}=\frac{(\rho c_p)_{eff}u}{k_{eff}}\frac{\partial T}{\partial x}+\frac{(\rho c_p)_{eff}}{k_{eff}}\frac{\partial T}{\partial t} \tag{18}$$

Adopting the adimensional variables below

$$X=\frac{x}{L},\quad \theta=\frac{T-T_a}{T_b-T_a},\quad \tau=\frac{k_{eff}t}{(\rho c_p)_{eff}L^2} \tag{19}$$

The controlling equation in Eq. (18) becomes

$$(1+4R)\frac{\partial^2\theta}{\partial X^2} - S_h\theta^2 - (Mc + Mr - Q_{\gamma e})\theta + Q_e = Pe_e\frac{\partial\theta}{\partial X} + \frac{\partial\theta}{\partial\tau} \tag{20}$$

where

$$Rd = \frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}},\quad Mc^2 = \frac{hP(1-\phi)L^2}{A_{cr}k_{eff}},\quad Mr = \frac{4\sigma\varepsilon P T_a^3 L^2}{A_{cr}k_{eff}},\quad Q_\gamma = \frac{\gamma(1-\phi)q_o''' L^2}{k_{eff}},$$
$$Q = \frac{(1-\phi)q_o''' L^2}{k_{eff}(T_b - T_a)},\quad Pe = \frac{(\rho c_p)_{eff}u_o L}{k_{eff}},\quad S_h = \frac{\rho_f c_{p,f} gK\beta W\phi L^2 (T_b - T_a)}{\nu A_{cr}k_{eff}} \tag{21}$$

Eq. (20) is alternative written as

$$\frac{\partial^2\theta}{\partial X^2} - S_p\theta^2 - (Nc + Nr - Q_\gamma)\theta + Q = Pe_R\frac{\partial\theta}{\partial X} + \zeta\frac{\partial\theta}{\partial\tau} \tag{22}$$

where

$$Nc = \frac{Mc}{1+4R},\quad Nr = \frac{Mr}{1+4R}\quad S_p = \frac{S_h}{1+4R},\quad Q_\gamma = \frac{Q_{\gamma e}}{1+4R},$$
$$Q = \frac{Q_e}{(1+4R)},\quad Pe_R = \frac{Pe_e}{(1+4R)},\quad \zeta = \frac{1}{(1+4R)}$$

Therefore, the adimensional initial condition is

$$\theta = \theta_0 \text{ when } \tau = 0,\ \text{ for } 0 < X < 1, \tag{23}$$

and, the dimensionless boundary conditions are given as

$$\theta = 1, \text{ at } X = 0 \text{ for } \tau > 0, \tag{24a}$$

$$\frac{\partial\theta}{\partial X} = 0, \text{ at } X = 1, \text{ for } \tau > 0, \tag{24b}$$

3. METHOD OF LINES

Method of lines is a semi-discretization method. The inherent simplicity of the computational procedures and superiority of stability advantage over the direct finite difference method has encouraged its various applications. [37-74].

3. 1. Method of Lines the Transient Thermal Problems

Applying method of lines to Eq. (22)

$$\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2}\right)-S_p\theta_i^2-\left(Nc+Nr-Q_\gamma\right)\theta_i+Q=Pe_R\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h}\right)+\zeta\frac{d\theta_i}{d\tau} \tag{25}$$

Collecting like terms

$$\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2}\right)-Pe_R\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h}\right)-S_p\theta_i^2-\left(Nc+Nr-Q_\gamma\right)\theta_i+Q=\zeta\frac{d\theta_i}{d\tau} \tag{26}$$

Which can be expressed as

$$\frac{d\theta_i}{d\tau}=\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2\zeta}\right)-Pe_R\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h\zeta}\right)-\frac{S_p}{\zeta}\theta_i^2-\left(\frac{Nc+Nr-Q_\gamma}{\zeta}\right)\theta_i+\frac{Q}{\zeta} \tag{27}$$

$i=1,2,...,N$

From the initial condition is

$$\theta_i=0,\quad \text{at } \tau=0,\ i=1,2,...,N \tag{28}$$

While for boundary conditions, we have

$$\frac{\theta_2(\tau)-\theta_0(\tau)}{2h}=0 \ \Rightarrow\ \theta_2(\tau)=\theta_0(\tau),\quad \rightarrow\theta_0(\tau)=\theta_0 \quad at \quad i=0 \tag{29a}$$

and

$$\theta_{N+1}(\tau)=1,\quad \text{at } i=N+1 \tag{29b}$$

where

N is the number of interior node points used in the discretization of the space, x $h=\frac{1}{N+1}$ is the node spacing or space step.

Which can be written as

$$\frac{d\theta_1}{d\tau} = \left(\frac{\theta_2 - 2\theta_1 + \theta_0}{h^2\zeta}\right) - Pe_R\left(\frac{\theta_2 - \theta_0}{2h\zeta}\right) - \frac{S_p}{\zeta}\theta_1^2 - \left(\frac{Nc + Nr - Q_\gamma}{\zeta}\right)\theta_1 + \frac{Q}{\zeta}$$

$$\frac{d\theta_2}{d\tau} = \left(\frac{\theta_3 - 2\theta_2 + \theta_1}{h^2\zeta}\right) - Pe_R\left(\frac{\theta_3 - \theta_1}{2h\zeta}\right) - \frac{S_p}{\zeta}\theta_2^2 - \left(\frac{Nc + Nr - Q_\gamma}{\zeta}\right)\theta_2 + \frac{Q}{\zeta}$$

$$\frac{d\theta_3}{d\tau} = \left(\frac{\theta_4 - 2\theta_3 + \theta_2}{h^2\zeta}\right) - Pe_R\left(\frac{\theta_3 - \theta_1}{2h\zeta}\right) - \frac{S_p}{\zeta}\theta_3^2 - \left(\frac{Nc + Nr - Q_\gamma}{\zeta}\right)\theta_3 + \frac{Q}{\zeta} \quad (30)$$

$$\frac{d\theta_N}{d\tau} = \left(\frac{\theta_{N+1} - 2\theta_N + \theta_{N-1}}{h^2\zeta}\right) - Pe_R\left(\frac{\theta_{N+1} - \theta_{N-1}}{2h\zeta}\right) - \frac{S_p}{\zeta}\theta_N^2 - \left(\frac{Nc + Nr - Q_\gamma}{\zeta}\right)\theta_N + \frac{Q}{\zeta}$$

The resulting ODE is generally of the form

$$\frac{d\theta_i}{dt} = A\theta_{i+1}^2 + B\theta_{i+1} + C\theta_i + D\theta_{i-1} + E \quad (31)$$

Eq. (18) is a system of N nonlinear first order differential equations and can be written in matrix form as

$$\frac{d\Theta}{dt} = A_M\Phi + A_L\Theta + K \quad (32)$$

where

$$A_M = A = \frac{S_p}{\zeta}\begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}$$

$$A_L = (B + C + D) = \frac{1}{h^2\zeta}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -2 \end{bmatrix} + \frac{Pe_R}{2h\zeta}\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$+\left(\frac{Nc + Nr - Q_\gamma}{\zeta}\right)\begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}$$

$$\bar{\Phi} = [\theta_1^2 \quad \theta_2^2 \quad \ldots \quad \theta_N^2]^T$$

$$\Theta = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_N]^T$$

and

b is a column vector of order $N \times 1$ which is in the form

$$E = \frac{Q}{\zeta}[1 \quad 1 \quad \ldots \quad 1 \quad 1]^T$$

Euler's method is used to solve the system of the nonlinear differential equations. The method is stated as follows

$$\theta_i^{n+1} = \theta_i^n + hf\left(\theta_i^n\right)$$

where $f\left(\theta_i^n\right) = A\theta_{i+1}^2 + B\theta_{i+1} + C\theta_i + D\theta_{i-1} + E$

h is the time-step

For the linear thermal model, the resulting ODE is generally of the form

$$\frac{d\theta_i}{dt} = B\theta_{i+1} + C\theta_i + D\theta_{i-1} + E \tag{33}$$

Also, Eq. (33) is a system of N linear first order differential equations and can be written in matrix form as

$$\frac{d\Theta}{dt} = A_L\Theta + b \tag{34}$$

where,

A_L is a $N \times N$ matrix given by

$$A_L = \frac{1}{h^2}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -2 \end{bmatrix}$$

$$(B+C+D) = \frac{1}{h^2\zeta}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -2 \end{bmatrix} + \frac{Pe_R}{2h\zeta}\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$
$$+\left(\frac{Nc+Nr-Q_\gamma}{\zeta}\right)\begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}$$

and

$$\Theta = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_N]^T$$

b is a column vector of order $N \times 1$ which is in the form

$$b = \frac{1}{h^2}[\theta_0 \quad 0 \quad \ldots \quad 0 \quad \theta_{N+1}]^T = \frac{1}{h^2}[\theta_0 \quad 0 \quad \ldots \quad 0 \quad 1]^T$$

Eigen value and Eigen vector method is used to solve the system of the linear differential equations

3. NUMERICAL SOLUTION OF THE NONLINEAR TRANSIENT THERMAL MODEL

$$\left(\frac{\theta_{i+1}^{n+1} - 2\theta_i^{n+1} + \theta_{i-1}^{n+1} + \theta_{i+1}^{n} - 2\theta_i^{n} + \theta_{i-1}^{n}}{2\Delta^2 X}\right) - S_p\left(\theta_i^n\right)^2 - \left(Nc + Nr - Q_\gamma\right)\theta_i^n + Q$$
$$= Pe_R\left(\frac{\theta_{i+1}^{n+1} - \theta_{i-1}^{n+1} + \theta_{i+1}^{n} - \theta_{i-1}^{n}}{4\Delta X}\right) + \zeta\left(\frac{\theta_i^{n+1} - \theta_i^n}{\Delta\tau}\right) \tag{35}$$

Which can be arranged as

$$\left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right)\theta_{i+1}^{n+1} - \left(\frac{2}{2\Delta^2 X} + \frac{\zeta}{\Delta\tau}\right)\theta_i^{n+1} + \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right)\theta_{i-1}^{n+1} + \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right)\theta_{i+1}^{n}$$
$$-\left(\frac{2}{2\Delta^2 X} - \frac{\zeta}{\Delta\tau} + \left(Nc + Nr - Q_\gamma\right)\right)\theta_i^n - S_p\left(\theta_i^n\right)^2 + \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right)\theta_{i-1}^{n} + Q = 0 \tag{36}$$

We can write Eq. (36) as

$$A\theta_{i+1}^{n+1} + B\theta_i^{n+1} + C\theta_{i-1}^{n+1} + D\theta_{i+1}^{n} + E\theta_i^{n} + F\left(\theta_i^n\right)^2 + G\theta_{i-1}^{n} + H = 0 \tag{37}$$

where

$$A = \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right), \quad B = -\left(\frac{2}{2\Delta^2 X} + \frac{\zeta}{\Delta\tau}\right), \quad C = \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right), \quad D = \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right),$$

$$E = -\left(\frac{2}{2\Delta^2 X} - \frac{\zeta}{\Delta\tau} + \left(Nc + Nr - Q_\gamma\right)\right), \quad F = -S_p, \quad G = \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right), \quad H = Q,$$

The finite difference discretization of the initial condition is

$$\theta_i^o = 0, \tag{38}$$

While FDM for boundary conditions become

$$\frac{\theta_1^n - \theta_{-1}^n}{2\Delta X} = 0 \quad \Rightarrow \quad \theta_1^n = \theta_{-1}^n \tag{39}$$

$$\theta_M^n = 1$$

4. RESULTS AND DISCUSSION

In order to establish the accuracy of the present method, we compared the results of the numerical method of lines (NMOL) with the results of finite difference method (FDM) as shown in Fig. 2. Having established this fact, the numerical solutions was then used to explored the impact of Peclet number on the thermal responses of the extended surfaces under various values of the controlling parameters.

Fig. 3 presents the effect of Peclet number on the thermal response of the fin when it is not moving and it is moving slowly. The Peclet number bears direct relationship with the fin temperature as depicted in the figure. This is due to the increase in the speed of the fin that reduces the material exposure time to the surrounding fluid. Therefore, low Peclet number favours cooling enhancement.

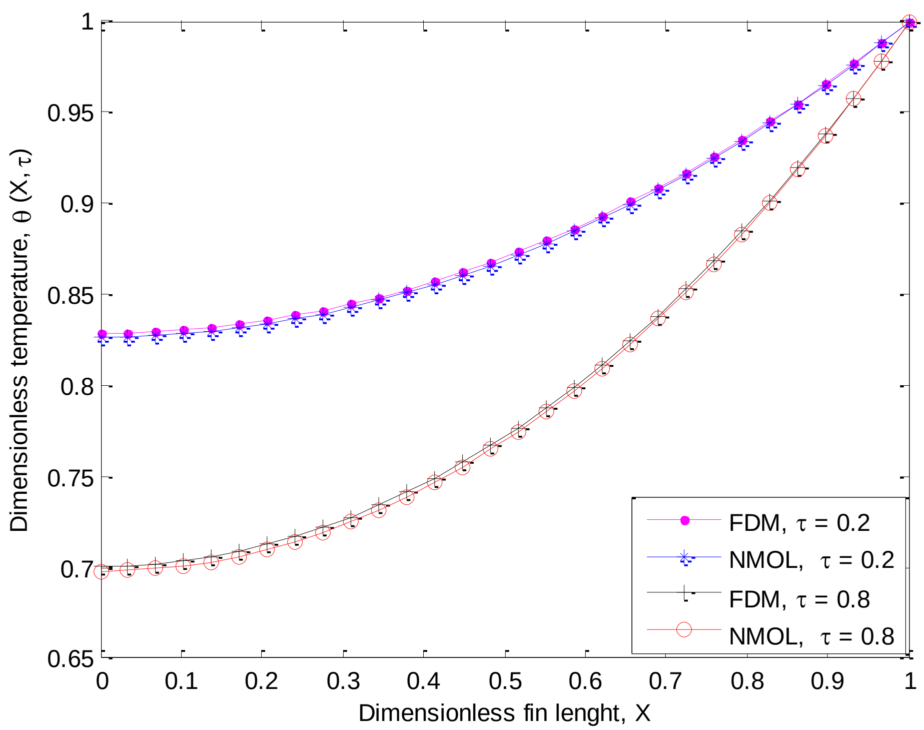

Fig. 2. Comparison of results of NMOL and FDM

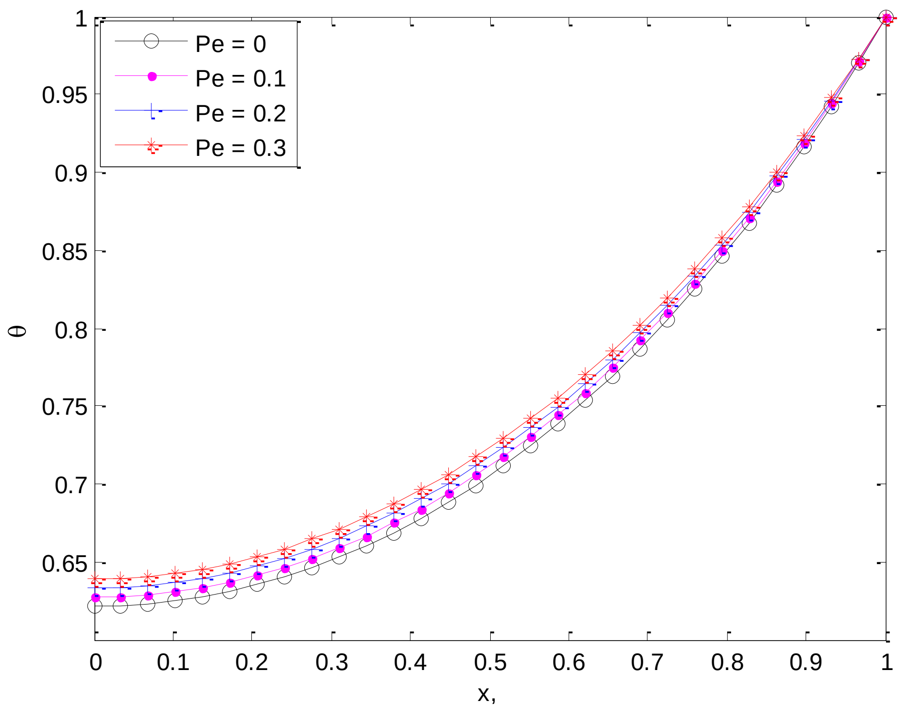

Fig. 3. Effect of Peclet number on the temperature when Nc=0.8, Nr=0.2, θ_a=0.1 θ_s=0.2

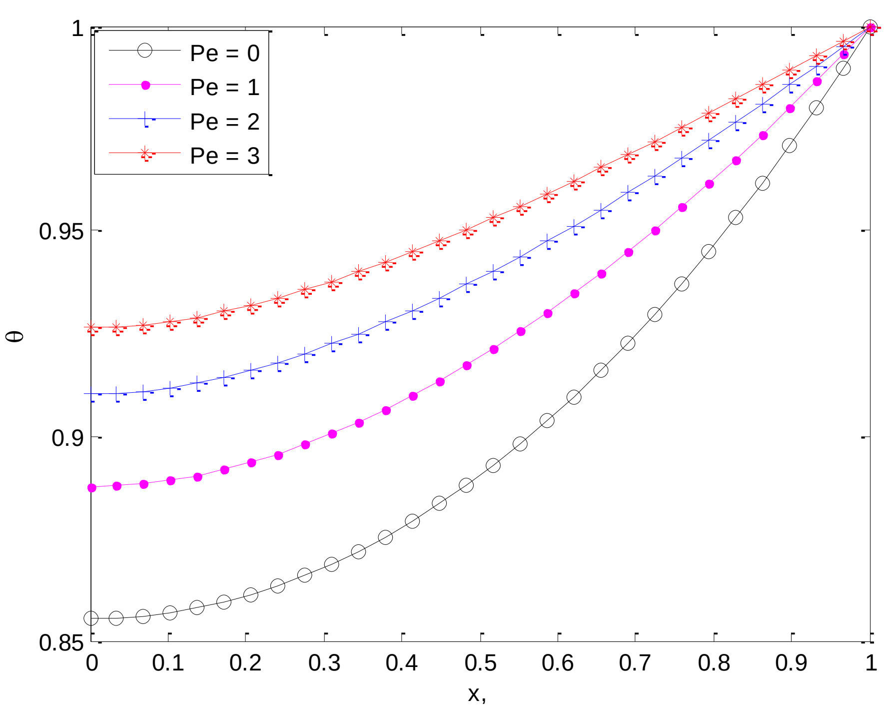

Fig. 4. Impact of Peclet number on the temperature when Nc=0.5, Nr=0.5 θ_a=0.3 θ_s=0.2

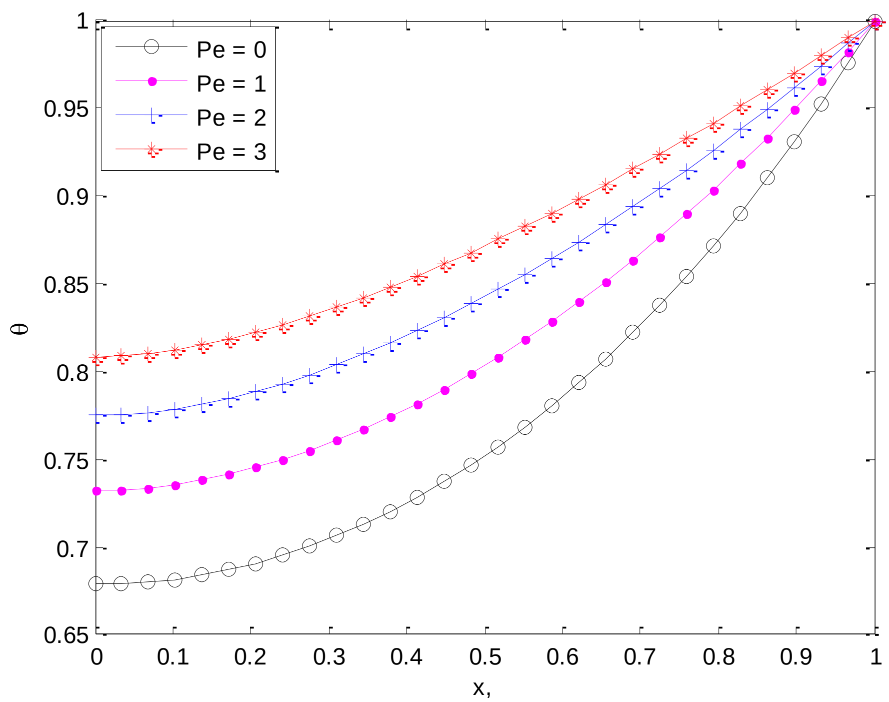

Fig. 5. Impact of Peclet number on the temperature when Nc=0.5, Nr=0.8 θ_a=0.2, θ_s=0.2

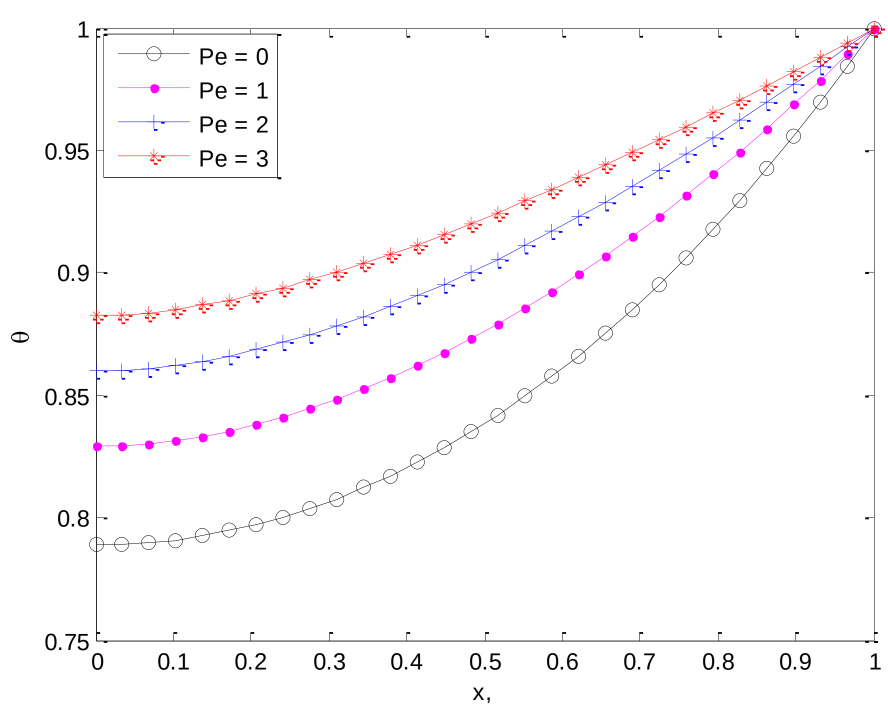

Fig. 6. Effect of Peclet number on the temperature when Nc=0.5, Nr=0.5 θ_a=0.2 θ_s=0.4

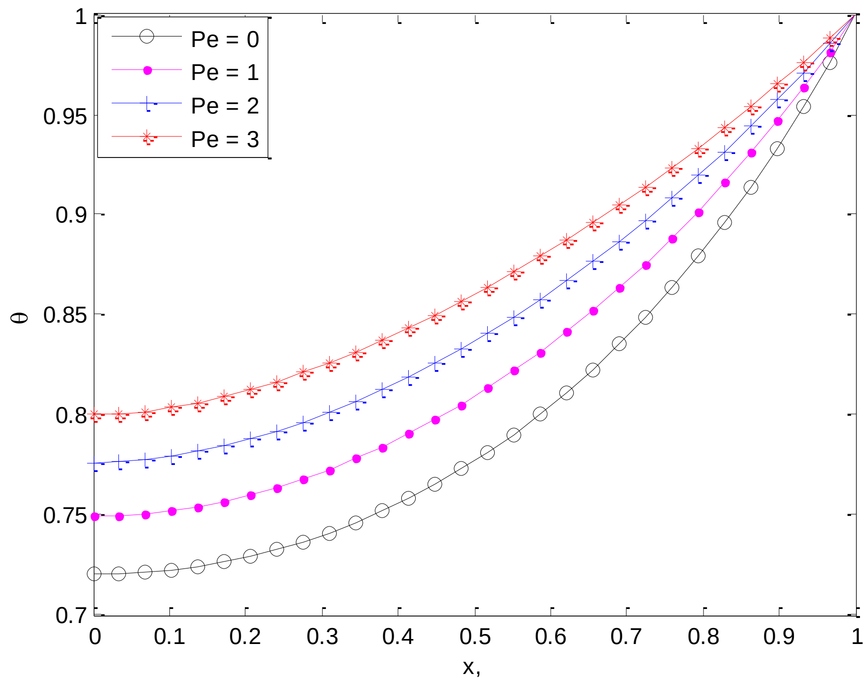

Fig. 7. Impact of Peclet number on the temperature when Nc=0.5, Nr=0.5, S_p =0.3 θ_a=0.2 θ_s=0.0

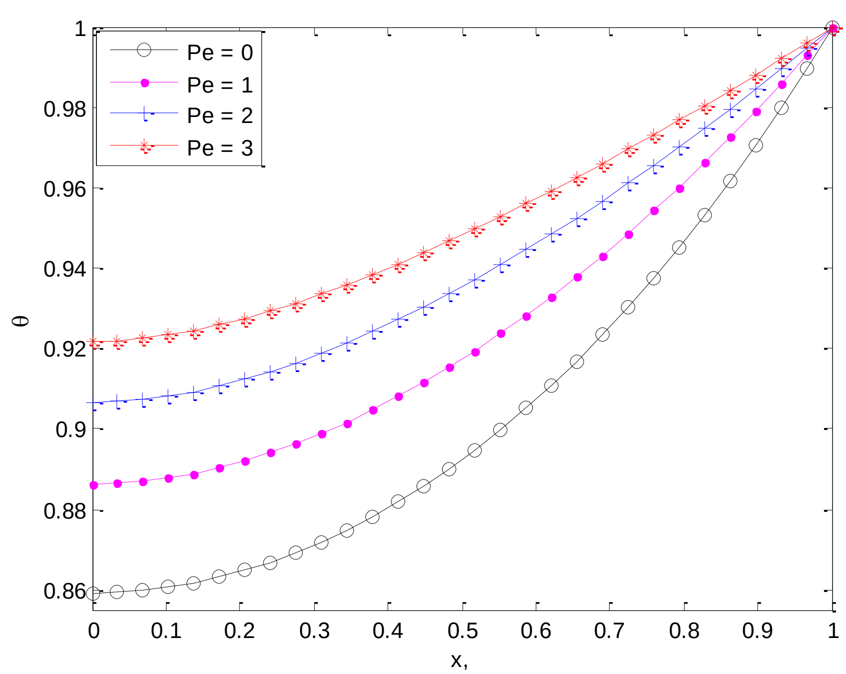

Fig. 8. Impact of Peclet number on the temperature when Nc=0.8 Nr=0.2, Q = 0.1, θ_a=0.2 θ_s=0.2

Figs. 4-8 illustrate the impact of Peclet number on the thermal responses of the extended surfaces under various values of the controlling parameters. The controlling parameters such as convective-conductive, radiative-conductive, ambient temperature, porosity and internal heat generation parameters have significant influence on the thermal behaviour of the porous moving fin. Closer investigation revealed that when the convective-conductive and radiative-conductive parameters increase, the fin temperature decreases. However, the fin Peclet number increases when ambient and surface temperatures increase while the temperature of the fin decreases as the convective-conductive, radiative-conductive and porous terms increase. Additionally, an increase in the internal heat generation causes the fin temperature to increase. Under all these various varying values of the controlling parameters, it is established that the temperature of the fin increases as the Peclet number increases as depicted in all the figures.

5. CONCLUSION

Numerical method of lines has been applied in this study to investigate the effect of Peclet number on the transient nonlinear thermal behaviour of moving porous fins. The results showed that the Peclet number increases with increasing fin temperature. The numerical investigations reveal that the fin temperature decreases as the convective-conductive, radiative-conductive and porous terms increase. However, when internal heat generation, ambient and surface temperatures increase, the fin Peclet number also increases. Under all these various varying values of the controlling parameters the temperature of the fin increases as the Peclet number increases. The physical insight depicted in this study will create a new path for the nonlinear analysis of the thermal problems.

References

[1] M. Torabi, H. Yaghoobi and A. Aziz Analytical Solution for Convective-Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity. *Int. J. Thermophysics* (2012) 33: 924-941

[2] A. Aziz, F. Khani, Convection-radiation from a continuously moving fin of variable thermal conductivity. *J. of Franklin Institute* 348 (2011) 640-651

[3] A. Aziz and R. J. Lopez, Convection -radiation from a continuously moving, variable thermal conductivity sheet or rod undergoing thermal processing, *I. J. of Thermal Sciences* 50 (2011) 1523-1531

[4] S. Singh, D. Kumar, K. N. Rai. Wavelet Collocation Solution for Convective-Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity. *International Journal of Engineering and Advanced Technology*, 2(4), 2013

[5] A. Aziz, M. Torabi, Covective-radiative fins with simultaneous variation of thermal conductivity, heat transfer coef ficient and surface emissivity with temperature, *Heat Transfer Asian Research* 41 (2) (2012)

[6] J. Ma, Y. Sun, B. W. Li, H. Chen Spectral collocation method for radiative–conductive porous fin with temperature dependent properties. *Energy Conversion and Management* 111 (2016) 279–288

[7] Y. Sun J. Ma, B. W. Li, H. Spectral collocation method for convective-radiative transfer of a moving rod with variable thermal conductivity. *International Journal of Thermal Sciences* 90 (2015) 187-196

[8] A.S.V. R. Kanth and N. U. Kumar. Application of the Haar Wavelet Method on a Continuously Moving Convective-Radiative Fin with Variable Thermal Conductivity. Heat Transfer - Asian Research. Volume 42, Issue 4, June 2013, Pages 335-351

[9] A. S. V. R. Kanth and N. U. Kumar. A Haar Wavelet Study on Convective-Radiative Fin under Continuous Motion with Temperature-Dependent Thermal Conductivity. *Walailak J Sci & Tech.* 11(3) (2014) 211-224

[10] R. K. Singla and D. Ranjan. Application of decomposition method and inverse parameters in a moving fin, *Energy Conversion and Management*, 84 (2014) 268-281

[11] A. Moradi and R. Rafiee. Analytical Solution to Convection-Radiation of a Continuously Moving Fin with Temperature-Dependent thermal conductivity, *Thermal Science*, 17 (2003) 1049-1060

[12] A. S. Dogonchi and D. D. Ganji. Convection-Radiation heat transfer study of moving fin with temperature dependent thermal conductivity, heat transfer coefficient and heat generation, *Applied Thermal Engineering*, 103(2016), 705-712

[13] Y. S. Sun and J. Ma. Application of Collocation Spectral Method to Solve a Convective – Radiative Longitudinal Fin with Temperature Dependent Internal Heat Generation, Thermal Conductivity and Heat Transfer Coefficient, *Journal of Computational and Theoretical Nano-Science*, 12 (2015) 2851-2860

[14] M. G. Sobamowo, A. A. Yinusa, M. O. Salami, O. C. Osih, B. O. Adesoye. Heat Transfer Analysis of a Rectangular Moving Porous Fin with Temperature-Dependent Thermal Conductivity and Internal Heat Generation: Comparative and Parametric Studies. *Engineering Advances.* 1(2) (2021) 50-66

[15] P. L. Ndlovu and R. J. Moitsheki. Analysis of temperature distribution in radial moving fins with temperature dependent thermal conductivity and heat transfer coefficient. *International Journal of Thermal Sciences* 145 (2019) 106-115

[16] P. L. Ndlovu and R. J. Moitsheki. A Study of Transient Heat Transfer through a Moving Fin with Temperature Dependent Thermal Properties. *Defect and Diffusion Forum* 401 (2020) 1-13

[17] P. L. Ndlovu and R. J. Moitsheki. Analysis of a Convective-Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity. International Journal of Nonlinear Sciences and Numerical Simulation. 21(3-4) (2020). https://doi.org/10.1515/ijnsns-2018-0206

[18] G. Sowmya, B. J. Gireesha, M. Madhu, Macha 2020. Analysis of a fully wetted moving fin with temperature-dependent internal heat generation using the finite element method. *Heat Transfer*, 49(4) (2020) 1939-1954

[19] B. J., Gireesha, G., Sowmya, S., Sindhu. Analysis of thermal behavior of moving longitudinal porous fin wetted with water-based SWCNTs and MWCNTs. *Heat Transfer*, 49(4) (2020) 2044-2058

[20] G. Sowmya, Bijjanal Jayanna Gireesha, Analysis of heat transfer through different profiled longitudinal porous fin by differential transformation method, *Heat Transfer*, 10.1002/htj.22394, 51, 2, (2165-2180), (2021)

[21] Gireesha, B.J., Sowmya, G. and Macha, M. (2022). Temperature distribution analysis in a fully wet moving radial porous fin by finite element method, *International Journal of Numerical Methods for Heat & Fluid Flow*, Vol. 32 No. 2, pp. 453-468. https://doi.org/10.1108/HFF-12-2018-0744

[22] F. M. Najafabadi, T. H. Rostami, K. Hosseinzadeh, D. D. Ganji. Thermal analysis of a moving fin using the radial basis function approximation. *Heat transfer,* 50(8) (2021) 7553-7567

[23] M. G. Sobamowo, A. A. Yinusa, O. P. Popoola, M. A. Waheed. Transient Thermal Analysis of Convective-Radiative Moving Fin under the Influences of Magnetic Field and Time-dependent Boundary Condition. *Computational Sciences and Engineering* 1(2) (2021) 139-152

[24] M. G. Sobamowo, M. O. Salami, A. A. Yinusa. Thermal Analysis of a Convective-Radiative Moving Porous Trapezoidal Fin with Variable Thermal Properties and Internal Heat Generation using Finite Element Method. *World Scientific News* 163 (2022) 139-157

[25] Nabati, M., Taherifar, S. & Jalalvand, M. Sinc–Galerkin approach for thermal analysis of moving porous fin subject to nanoliquid flow with different shaped nanoparticles. *Math Sci* (2021). https://doi.org/10.1007/s40096-021-00387-4

[26] Gireesha B, Sowmya G, Gorla RSR. Nanoparticle shape effect on the thermal behaviour of moving longitudinal porous fin. *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*. 2020; 234(3-4): 115-121. doi:10.1177/2397791420915139

[27] B. J. Gireesha, G. Sowmya, M. I. Khan, et al. Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection. *Comput Meth Prog Bio* 2019; 185: 105166

[28] M. Turkyilmazoglu. Efficiency of the longitudinal fins of trapezoidal profile in motion. *Journal of Heat Transfer.* 2017, 30(4) (2020) 1867-1879

[29] M. G. Sobamowo and M. A. Waheed. Analytical investigation of heat transfer in a moving convective porous fin with temperature dependent thermal conductivity and internal heat generation. *World Scientific News* 150 (2022) 1-21

[30] G. A. Oguntala, M. G. Sobamowo, A. A. Yinusa and R. ABD-Alhameed. Determination of Proper Fin Length of a Convective-Radiative Moving Fin of Functionally Graded Material Subjected to Lorentz Force. *Defect and Diffusion Forum*, 401 (2020) 14-24

[31] B.J. Gireesha, G. Sowmya, M. Ijaz Khan, Hakan F. Öztop, Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection, Computer Methods and Programs in Biomedicine, Volume 185, 2020, 105166, https://doi.org/10.1016/j.cmpb.2019.105166

[32] M. G. Sobamowo, O. M. Kamiyo, A. A. Yinusa, and M. O. Salami. Exploration of the effects of fin geometry and material properties on thermal performance of convective-radiative moving fins. *Eng. Appl. Sci. Lett.* 2019, 2(3), 14-29

[33] M. G. Sobamowo, O. M. Kamiyo, M. O. Salami, A. A. Yinusa. Thermal assessment of a convective porous moving fins of different material properties using Laplace-variational iterative method. *World Scientific News* 139(2) (2020) 135-154.

[34] A. Jemiseye, M. G. Sobamowo and O. Olayiwola. Transient Thermal Cooling of Electronics Systems using Functional Graded Fins: Hybrid Computational Analysis. The Journal of Engineering and Exact Sciences 9(4):1-15. 10.18540/jcecvl9iss4pp15810-01e

[35] M. G. Sobamowo, J. N. Ojuro, O. K. Onanuga, A. M. O. Siqueira and J. C. C. Campos. Explicit Exact Solutions of Nonlinear Transient Thermal Models of a Porous Moving Fin using Laplace transform - Exp-function method. *The Journal of Engineering and Exact Sciences*. 09(10), (2023) 1-17, DOI:10.18540/jcecvl9iss10pp15972-01e

[36] Y. Jaluria, A.P. Singh, Temperature distribution in a moving material subjected to surface energy transfer, *Computer Methods in Applied Mechanics and* Engineering 41 (1983) 145–156

[37] Y. Jaluria. Transport from continuously moving materials undergoing thermal processing, *Annual Reviews of Heat Transfer*, vol. 4, Academic Press, 1992, pp. 187–245 (Chapter Four).

[38] O. A. Liskovets. The method of lines (Review), *Differential Equations*, 1 (1965), 1308 - 1323. (English version)

[39] L. S. Xanthis, The numerical method of lines and ODE solvers can provide a new powerful computational fracture mechanics tool, *Int. Conference on Computational Mechanics,* Tokyo, Japan (1986).

[40] T. Shou-gao. And Y. Si. The vibration and stability analysis of moderate thick plates by the method of lines. *Applied Mathematics and Mechanics* 13(6) (1992).

[41] J. G. Verwer and J. M. Sanz-Serna. (1984). Convergence of method of lines approximations to partial differential equations, *Computing,* 33(1984), 297-313

[42] S. C. Reddy and L. N. Trefethen. Stability of the method of lines, *Numerische Mathematik*, 62(1)(1992), 235-267

[43] A. Zafarullah. Application of the method of lines to parabolic partial differential equations with error estimates, *Journal of the Association for Computing Machinery*, 17 (1970), 294-302

[44] A. Campo and M. Arıcı. Semi-analytical, piecewise temperature–time distributions in solid bodies of regular shape affected by uniform surface heat flux: Combination of the

Method of Lines (MOL) and the eigenvalue method, *International Communications in Heat and Mass Transfer*, 108 (2019) 104276

[45] Campo A. The Numerical Method of Lines facilitates the instruction of unsteady heat conduction in simple solid bodies with convective surfaces. *International Journal of Mechanical Engineering Education* 2022; 50(1): 3-19. doi:10.1177/0306419020910423

[46] A. Campo and J. Garza. Transversal Method of Lines (TMOL) for unsteady heat conduction with uniform surface heat flux, *ASME Journal of Heat Transfer*, 136 (2014), 111302

[47] A. Campo and Y. Masip-Macía. Semi-analytical solution of unsteady heat conduction in a large plane wall with convective boundary conditions for the "small-time" sub-domain using the Transversal Method of Lines (TMOL), *International Journal of Numerical Methods in Heat and Fluid Flow,* 29(2) (2019) 536-552

[48] A. Campo and J. Sieres. Semi-analytical treatment of the unsteady heat conduction equation with prescribed surface temperature: The Transversal Method Of Lines (TMOL) delimited to the "small time" sub-domain, *International Communications in Heat and Mass Transfer,* 116(2020). Article number 1046872020

[49] A. Campo and D. J. Celentano. Improved Transversal Method of Lines (ITMOL) for unidirectional, un-steady heat conduction in regular solid bodies with heat convection exchange to nearby fluids, *Computational Thermal Sciences: An International Journal*, 12(2), 179-189, 2020

[50] S. S. Hu and W. E. Schiesser, An adaptive grid method in the numerical method of lines, in Advances in Computer Methods for Partial Differential Equations, R. Vichnevetsky and R. Stepleman, Eds., 1981, 305–311

[51] W. E. Schiesser, The Numerical Method of Lines: Integration of Partial Differential Equations, Academic Press, San Diego, CA, USA, 1991.

[52] S. Hamdi, W. H. Enright, Y. Ouellet, and W. E. Schiesser, Method of lines solutions of the extended Boussinesq equations, *Journal of Computational and Applied Mathematics*, 183(2) (2005) 327-342

[53] P. Saucez, A. Vande Wouwer, W. E. Schiesser, and P. Zegeling, Method of lines study of nonlinear dispersive waves, Journal of Computational and Applied Mathematics, 168(1-2) (2004), 1-2, 413–423

[54] J. G. Verwer and J. M. Sanz-Serna. Convergence of method of lines approximations to partial differential equations, *Computing,* 33(184) 297-313

[55] Schiesser, W.E. The Numerical Method of Lines: Integration of Partial Differential Equations. 2nd Edition, Clarendon Presses, Oxford, 1978.

[56] Sharaf, A.A. and Bakodah, H.O. A Good Spatial Discretization in the Method of Lines. *Applied Mathematics and Computation,* 171-172 (2005) 1253-1263

[57] M. B. Carver and H. W. Hinds. (1978) The Method of lines and the Advective Equation. *Simulation*, 31 (1978) 59-69

[58] H. Han and Z. Huang, The Direct Method of Lines for Numerical Solution of Interface Problem, *Comput. Mehtods Appl. Mech. Engrg.* 171, (1999) 61-75

[59] A. Ismail, F. Karim, G.D. Roy and M. Ashaque, Numerical Modeling of Tsunami Via The Method of Lines, Word *Academy of Science, Engineering and Technology,* 32(35) (2007) 177-185

[60] M.J. Al-Majid, Method of Lines Analysis of Gaussian Beam Coupling to Dielectric Slab Waveguide, M.Sc. Thesis in Electrical Engineering, University of Petroleum & Minerals Dhahran, Saudi Arabia, 1994.

[61] R. Celi, State-Space Representation of Vortex Wakes by The Method of Lines, *J. of the American Helicopter Society,* 50(2) (2005) 195-205

[62] N.K. Madsen, The Method of Lines for The Numerical Solution of Partial Differential Equations, *ACM SIGNUM Newsletter.* 10(10) (1975) 5-7

[63] B.J. Noye and H. H. Tan, Finite Difference Methods for Solving Two-Dimensional Advection-Diffusion Equation, *Int. J. num. Meths. in Fluid,* 9(1989) 75-98

[64] A. Özdes and E. N. Aksan, The Method of Lines Solution of The Korteweg-de Vries Equation for Small Times, *Int. J. Contemp. Math. Sciences* 1, 13 (2006) 639-650

[65] S. Pamuk and A. Erdem, The Method of Lines for The Numerical Solution of A Mathematical Model for Capillary Formation: The Role of Endothelial Cells in The Capillary, *Appl. Math. and Comput.* 186 (2007),831-835

[66] H. Han and Z. Huang, The Direct Method of Lines for Numerical Solution of Interface Problem, *Comput. Mehtods Appl. Mech. Eng* 171(1999) 61-75

[67] A. Ismail, F. Karim, G.D. Roy and M. Ashaque, Numerical Modeling of Tsunami Via The Method of Lines, *Word Academy of Science, Engineering and Technology,* 32(35) (2007) 177-185

[68] M. Javidi, A Numerical Solution of Burger's Equation Based on Modified Extended BDF Scheme, *Int. Math. Forum,* 1(32) (2006) 1565-1570

[69] J.-P. Kauthen, The Method of Lines for Parabolic Partial Integro-Differential Equations, *J. of Integral Equations and Applications,* 4(1) (1992), 69-81

[70] Voigt, A., Line Method Approximations to Cauchy Problem for Nonlinear Parabolic Differential Equations, *Numer. Math.* 23 (1974) 23-36

[71] R. C. Thompson. Convergence and Error Estimates for the Method of Lines for Certain Nonlinear Elliptic and Elliptic-Parabolic Equations, *SIAM J. Numer. Anal.* 13(l), 1976, 27-43

[72] A. M. Loeb and W. E. Schiesser. Stiffness and Accuracy in the Method of Lines Integration of Partial Differential Equations. Stiff Differential Systems, ed. R. A. Willoughby, Plenum Press, New York, 1974, pp. 229-243.

[73] N. K. Madsen and R. E. Sincovec. The Numerical Method of Lines for the Solution of Nonlinear Partial Differential Equations, Lawrence Livermore Laboratory UCRL-75142, September 1973.

[74] V. Agarwal, G. Chaturvedi, A. Agrawal. Modern Mathematical Techniques for Analysis of Temperature Distribution in Straight Profile Fins. *International Journal of Engineering Research & Technology* 3(3), 2156-2160, 2014