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The temperature of electromagnetic waves and bounds for wavelengths of electromagnetic waves

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ABSTRACT

Every material becomes energy after it reaches certain temperature, and there is a need to define a concept of temperature for electromagnetic waves. It is fixed in this article such that the Wien's displacement law provides a most successful definition for temperature of electromagnetic waves. Some other displacement laws are derived in this article. The reasons for choosing the Wien's displacement law as the best displacement law for defining temperature of electromagnetic waves are explained. Moreover upper bound and lower bound for wavelengths of electromagnetic waves are obtained.

***Keywords*:** Planck's distribution law, Wien's displacement law, Planck's constant, Electromagnetic waves

1. INTRODUCTION

It was observed in the article [18] that every material begins to radiate, when its temperature is raised to 40541400000 Kelvin. So, there is a need to define a concept of temperature of an electromagnetic wave. First effort was taken in the article [19] by means of the Stefan Boltzmann law. Second effort was taken in the article entitled "Mean displacement law for black body radiations and temperature of black body radiations" appeared in the journal

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"Discovery" in Volume 59, by means of the Wien's displacement law and a mean displacement law. For this purpose, mean of the Planck's distribution was defined, it was observed that the mean satisfied a displacement law, and one temperature for electromagnetic waves was defined. Another temperature was defined in terms of the Wien's displacement law. It was difficult to choose the most suitable concept from these two temperatures for electromagnetic waves.

The present article fixes this problem, and declares the concept of temperature defined through the Wien's displacement law as the best one. This article also provides a class of displacement laws derivable from Planck's distribution.

The basic units which will be used in this article are metre, kilogram, second and Kelvin, and all derived units will also be replaced by these basic units. Let us consider the following constants with this fixation of units.

Planck's constant (for light waves) $h = 6.626 \times 10^{-34}$.

Boltzmann constant $k = 1.38 \times 10^{-23}$.

Speed of light in vacuum $c = 2.9979 \times 10^{8}$.

Wien universal constant $b = 2.8978 \times 10^{-3}$.

Let us consider a black body with temperature T, let us use the notation λ for wavelength of a radiated wave radiated from that black body, let us use the notation $u(\lambda)$ for intensity of the radiated wave with wavelength λ, and let us use the following form of the Planck's distribution [13, 21]:

$$u(\lambda) = \frac{8\pi hc}{\lambda^5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1} .$$

Let us use concepts introduced in the article [19]. Let us first recall the definition of an electromagnetic wave from the article [19]. An energy wave that can travel in vacuum with speed c, speed of light, is defined as an electromagnetic wave. Let us follow only this definition; not the other definitions. All light rays [12], all waves radiated from a black body, X-rays, gamma rays, radio waves, and gravitational waves are electromagnetic waves. See the article [19] for concepts and facts which are not presented in the present article. Alpha rays, beta rays, and electron rays from magnetrons are not electromagnetic waves.

There is a fundamental difference between the electromagnetic waves "light rays" and the electromagnetic waves "radio waves". Electromagnetic waves which emerge from electric fields are called electric field waves and electromagnetic waves which emerge from magnetic fields are called magnetic fields waves. Electric field waves contain only electric field component, and they do not contain magnetic field component. Magnetic field waves contain only magnetic field component, and they do not contain electric field component. Light rays are electric field waves and they do not contain magnetic field component. Radio waves are magnetic field waves and they do not contain electric field component.

These facts do not contradict Maxwell's equations. "Temperature" was defined in the previous works first for electric field waves and then it was defined for magnetic field waves, and thereby it was defined for all electromagnetic waves. The main purpose of the present article is to justify that "temperature" defined by using the Wien's displacement law is the best one. One more purpose is to find some displacement laws. Another purpose is to find an upper bound and a lower bound for wavelengths of electric field waves, magnetic field waves and electromagnetic waves.

2. DISPLACEMENT LAWS

This section is devoted only for black body radiations. Let us consider the Planck's distribution $u(\lambda)=\frac{8\pi hc}{\lambda^5}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}$. Let us first define the concept of displacement laws. A law that is in the form "$\lambda T = a\ constant$" will be called a displacement law. In this form "a constant" is a fixed constant, and λ and T are variables. For every constant in the right hand side, there is a displacement law. But, it need not be interesting.

This one becomes an interesting law only when there is another important property for the wavelengths λ satisfying the law. Let us first discuss the well known Wien's displacement law [1, 5], and let us conclude this one as an "interesting" law.

On solving the equation $\frac{du(\lambda)}{d\lambda}=0$, the solution obtained is $\lambda=\frac{b}{T}$, in which b is the universal Wien's constant, T is considered as a fixed constant at present, and λ is considered as a variable at present. It can be verified that $\frac{d^2u(\lambda)}{du^2}<0$ at this solution value $\lambda=\frac{b}{T}$. That is, each solution provides a maximum for the Planck's distribution whenever T is fixed.

That is, each solution provides a mode of the distribution $u(\lambda)$, whenever T is fixed. That is, the mode values $\lambda_T=\lambda=\frac{b}{T}$ satisfy a relation $\lambda_T T=b$, which is of the form $\lambda T=b$, in which both λ and T are varied subject to that relation. So, if the "constant" in the relation $\lambda T = a\ constant$ is chosen as the Wien's constant, then the corresponding displacement law $\lambda T=b$ is an interesting law, because the variable λ is the mode of the Planck's distribution corresponding to the temperature T.

Let us now discuss another interesting displacement law which was mentioned as "mean displacement law" in the article "Mean displacement law for black body radiations and temperature of black body radiations". The mean of the Planck distribution is given by $\frac{\int_0^\infty \lambda\, u(\lambda)d\lambda}{\int_0^\infty u(\lambda)d\lambda}$. When T is fixed, this mean value was found in that article as $\lambda_T = 5.3288843\times10^{-3}\times\frac{1}{T}$. This leads to the mean displacement law $\lambda T=5.3288843\times10^{-3}$.

This is another interesting displacement law, because the variable λ is the mean of the Planck's distribution corresponding to the temperature T. Let us give details of the derivation in a general way so that more displacement laws are derived.

For this purpose, let us begin with the known formula: $\int_0^\infty \frac{x^{s-1}}{e^x-1}\,dx=\zeta(s)\Gamma(s)$. Here $\zeta(s)$ is the standard Riemann zeta function [23] and $\Gamma(s)$ is the standard gamma function. Let us assume the formula for $s>1$. Let us first fix s at present. Let us consider the ratio $\frac{\int_0^\infty \lambda^s\, u(\lambda)d\lambda}{\int_0^\infty \lambda^{s-1}\, u(\lambda)d\lambda}$. Let us consider a common substitution $x=\frac{hc}{\lambda kT}$ or $\lambda=\frac{hc}{xkT}$ so that $dx=-\frac{hc}{kT\lambda^2}\,d\lambda$ or $d\lambda=-\frac{kT\lambda^2}{hc}\,dx=-\frac{hc}{kTx^2}\,dx$.

This common substitution leads to

$$\frac{\int_{\lambda=0}^{\infty}\lambda^{s-5}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}d\lambda}{\int_{\lambda=0}^{\infty}\lambda^{s-6}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}d\lambda}=\frac{hc}{kT}\frac{\int_{x=\infty}^{0}x^{3-s}(e^{x}-1)^{-1}dx}{\int_{x=\infty}^{0}x^{4-s}(e^{x}-1)^{-1}dx}$$
$$=\frac{hc}{kT}\frac{\zeta(4-s)\Gamma(4-s)}{\zeta(5-s)\Gamma(5-s)}$$

when $4-s>1$, or when $s<3$. So, this leads to an interesting displacement law

$$\lambda T=\frac{hc}{k}\frac{\zeta(4-s)\Gamma(4-s)}{\zeta(5-s)\Gamma(5-s)}$$

in which λ assumes the ratio value $\frac{\int_0^{\infty}\lambda^{s}\,u(\lambda)d\lambda}{\int_0^{\infty}\lambda^{s-1}\,u(\lambda)d\lambda}$ for which $T$is fixed, provided $s<3$. This one provides infinitely many interesting displacement laws. The mean displacement law can be derived for the case $s=1(<3)$. By using the values $\zeta(3)=1.2020569, \zeta(4)=1.0823232, \Gamma(3)=2!=2, \Gamma(4)=3!=6$, the mean displacement law can be derived as $\lambda T=5.3288843\times10^{-3}$. It is possible to derive more interesting displacement laws. Let us consider the ratio $\frac{\int_0^{\infty}\lambda^{s}\,u(\lambda)d\lambda}{\int_0^{\infty}\lambda^{s-2}\,u(\lambda)d\lambda}$. Let us consider a common substitution $x=\frac{hc}{\lambda kT}$ or $\lambda=\frac{hc}{xkT}$ so that $dx=-\frac{hc}{kT\lambda^2}\,d\lambda$ or $d\lambda=-\frac{kT\lambda^2}{hc}\,dx=-\frac{hc}{kTx^2}\,dx$. This common substitution leads to

$$\frac{\int_{\lambda=0}^{\infty}\lambda^{s-5}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}d\lambda}{\int_{\lambda=0}^{\infty}\lambda^{s-7}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}d\lambda}=\left(\frac{hc}{kT}\right)^{2}\frac{\int_{x=\infty}^{0}x^{3-s}(e^{x}-1)^{-1}dx}{\int_{x=\infty}^{0}x^{6-s}(e^{x}-1)^{-1}dx}$$
$$=\left(\frac{hc}{kT}\right)^{2}\frac{\zeta(4-s)\Gamma(4-s)}{\zeta(6-s)\Gamma(6-s)}$$

when $4-s>1$, or when $s<3$.

So, this leads to an interesting displacement law

$$\lambda T=\sqrt{\frac{hc}{k}\frac{\zeta(4-s)\Gamma(4-s)}{\zeta(6-s)\Gamma(6-s)}}$$

in which λ assumes the root value $\sqrt{\frac{\int_0^{\infty}\lambda^{s}\,u(\lambda)d\lambda}{\int_0^{\infty}\lambda^{s-2}\,u(\lambda)d\lambda}}$. One may work on other roots like cubic root, and then it is possible to derive more interesting displacement formulas.

It is also possible to derive two approximate displacement laws from [18] in the form $\lambda T\approx a\ constant$. Two approximate real roots of the equation $\frac{d^2u(\lambda)}{d\lambda^2}=0$ are $1.70356615852494\times10^{-3}$ and $4.05413975451854\times10^{-3}$. So, two interesting approximate displacement laws are $\lambda T \approx 1.70356615852494 \times 10^{-3}$ and $\lambda T \approx 4.05413975451854 \times 10^{-3}$. They are interesting in the view of solutions of the equation $\frac{d^2u(\lambda)}{d\lambda^2} = 0$.

One particular observation which will be used is that the average of the constants $1.70356615852494 \times 10^{-3}$ and $4.05413975451854 \times 10^{-3}$ is the Wien's constant b.

3. PLANCK'S CONSTANTS AND COSMIC MICROWAVE BACKGROUND

An interpretation for Planck's constant h comes through the Planck's equation $E = h\nu$, which are meant for light waves [10, 27]. There is a need to define Planck's constant ⓗ for radio waves, for which there is a need to know about minimum temperature of cosmic microwave background. After a very long period, and after an almost saturation of radiation, there is a natural minimum temperature in our universe. It is being a continuous effort to find this minimum temperature called temperature of cosmic microwave background [7, 8, 9, 25].

It is approximately 2.726 Kelvin. This temperature is essential to find Planck's constant for radio waves. An earlier theory establishes existence of a constant Λ such that any electromagnetic wave with wavelength $\lambda < \Lambda$ should be an electric field wave and any electromagnetic wave with wavelength $\lambda > \Lambda$ should be a magnetic field wave. On the other hand, every electric field wave with wavelength λ, it should be true that $\lambda \leq \Lambda$, and every magnetic field wave with wavelength λ, it should be true that $\lambda \geq \Lambda$. Such a number Λ should be unique. How to find it?

Suppose let us give an importance to the mode value λ_{max} of the Planck's distribution to represent the entire distribution, and let us consider the Wien's displacement law $\lambda_{max}T = b$. Let us consider this relation for $T = 2.726$ Kelvin. Then $\lambda_{max} = 1.063022744 \times 10^{-3}$ m.

This provides a representation for the entire distribution when radiation is about to stop and when the minimum temperature of cosmic microwave background is reached. Thus, any electromagnetic wave with wavelength $\lambda < 1.063022744 \times 10^{-3}$ should be an electric field wave and any electromagnetic wave with wavelength $\lambda > 1.063022744 \times 10^{-3}$ should be a magnetic field wave; when an importance is given to the mode value of the Planck's distribution to represent the entire distribution.

Why should importance not be given to the mean of Planck's distribution to represent the entire distribution? Let us derive a consequence when an importance is given to the mean of the Planck's distribution. The mean displacement law is $\lambda_{avg}T = 5.3288843 \times 10^{-3}$. Let us consider this relation for $T = 2.726$ Kelvin. Then $\lambda_{avg} = 1.9548365 \times 10^{-3}$ m. Thus, any electromagnetic wave with wavelength $\lambda < 1.9548365 \times 10^{-3}$ should be an electric field wave and any electromagnetic wave with wavelength $\lambda > 1.9548365 \times 10^{-3}$ should be a magnetic field wave; when an importance is given to the mean value of the Planck's distribution to represent the entire distribution.

The mode of the Planck's distribution gives the value $\Lambda = 1.063022744 \times 10^{-3}$, and the mean of the Planck's distribution gives the value $\Lambda = 1.9548365 \times 10^{-3}$. Only one of these two can be correct, when only mean and mode are the meaningful representations for the entire Planck's distribution. Authors could not find any reference for constructed/used light waves with wavelength greater than $1.063022744 \times 10^{-3}$ m. Moreover, there is an article in the title "200 GHz Maximum Oscillation Frequency in CVD Graphene Radio Frequency Transistors" which appeared in 2016, and which considers magnetic field waves with wavelength 1.5×10^{-3} m. So, there is a logical reason to accept the value $\Lambda = 1.063022744 \times 10^{-3}$ and to reject the value $\Lambda = 1.9548365 \times 10^{-3}$. Hereafter the value $\Lambda = 1.063022744 \times 10^{-3}$ will be used for further derivations.

So, any electromagnetic wave with wavelength $\lambda < 1.063022744 \times 10^{-3}$ m should be an electric field wave and any electromagnetic wave with wavelength $\lambda > 1.063022744 \times 10^{-3}$ m should be a magnetic field wave.

Now, let us derive the Planck's equation for magnetic field waves in the form $E = ⓗ\lambda$. Let us recall that the Planck's equation for electric field waves is $E = h\nu = \frac{hc}{\lambda}$. For the common boundary point $\lambda = 1.063022744 \times 10^{-3}$ between the interval of wavelengths of electric field waves and the interval of wavelengths of magnetic field waves, it should be true that $\frac{hc}{1.063022744 \times 10^{-3}} = E = ⓗ \times 1.063022744 \times 10^{-3}$. Then the value of the Planck's constant ⓗ for magnetic field waves should be ⓗ $= 17.57889908 \times 10^{-20}$. The factor 10^{-20} was not included in the article [19], which led to some miscalculations. The Planck's equation for magnetic field waves is $E = (17.57889908 \times 10^{-20})\lambda$. This one has been obtained by giving importance to the mode of the Planck's distribution for representation of the distribution.

4. TEMPERATURE OF ELECTROMAGNETIC WAVES

If temperature of electromagnetic waves is defined only by giving importance to the mode value, then it can be a meaningful definition. So, let us assume that $\Lambda = 1.063022744 \times 10^{-3}$ and let us assume that ⓗ $= 17.57889908 \times 10^{-20}$. Let us begin with an electric field wave with wavelength λ, and let us use the notation T_λ for the temperature of the electric field wave, which is to be defined. On converting the Wien's displacement law, to define T_λ, it is obtained that $\lambda T_\lambda = b$ and $T_\lambda = \frac{2.8978 \times 10^{-3}}{\lambda}$. Thus, the temperature T_λ of an electric field wave with wavelength λ is defined by the formula

$$T_\lambda = \frac{2.8978 \times 10^{-3}}{\lambda}.$$

This formula is obtained with an understanding that all wavelengths in the Planck's distribution of a black body with temperature T_λ are represented by a single wavelength, namely, the mode wavelength. Now, let us define temperature of a magnetic field wave. Let us consider a magnetic field wave with wavelength λ_m. Let us find a number λ_e by using the relation $\frac{hc}{\lambda_e} = ⓗ\lambda_m$ so that energy for an electric field wave with wavelength λ_e is equal to the energy of the magnetic field wave with wavelength λ_m.

That is, let us use the relation $\lambda_e = \frac{1.129996 \times 10^{-6}}{\lambda_m}$. Since energies are equal, it can be assumed that the temperatures are also equal. That is, $T_{\lambda_m} = T_{\lambda_e} = 2.8978 \times 10^{-3} \times \frac{1}{\lambda_e}$, when $\lambda_e = \frac{1.129996 \times 10^{-6}}{\lambda_m}$. That is, $T_{\lambda_m} = 2.564434 \times 10^{3} \times \lambda_m$. So, the temperature of a magnetic field wave with wavelength λ is defined by the formula

$$T_\lambda = 2.564434 \times 10^3 \times \lambda \,.$$

This formula has been derived from the formula for temperature of electric field waves which depends on the Wien's displacement law, and depends on ⓗ, which also depends on the Wien's displacement law. Thus, there are two formulas for temperature of electromagnetic waves, which depend on the Wien's displacement law, or equivalently, on the mode value of the Planck's distribution. One formula is for electric field waves and another one formula is for magnetic field waves. For electric field waves, temperature increases as wavelength decreases. For magnetic field waves, temperature increases as wavelength increases. Every interesting displacement law can be used to define a temperature, but the Wien's displacement law is considered as the best one for the reasons stated in the previous section, and for another reason that the average of the two distinct real roots of $\frac{d^2u(\lambda)}{d\lambda^2} = 0$ is the Wien's constant.

Temperature of electromagnetic waves is an important concept in view of the research articles [11, 20] which are concerned about heat generated on bodies by electromagnetic waves. When electrons move, magnetic fields are created. When blood (or a fluid) cells move, the electrons in the cells also move, and this movement creates magnetic fields. All living beings (with fluid flows) do have magnetic fields. When magnetic field waves disturb magnetic fields, heat may be produced.

All magnetic field waves are harmful to all living beings. Temperature of a magnetic field wave is increased when its wavelength is increased. More specifically, magnetic field waves with high wavelengths are more harmful to living beings than magnetic field waves with low wavelengths. Gravitational waves have very high wavelengths. It is stated in the introduction part of the article [26] that gravitational waves affect magnetic fields. Gravitational waves are produced from magnetic fields when two mega size stars come closer and their magnetic lines intersect. That is, gravitational waves are magnetic field waves which have speed c in vacuum. Power cables which are parallel to the metal arms of LIGOs create magnetic fields around the arms, and they are disturbed by gravitational waves causing changes in lengths of the arms [4, 17].

Every material has electrons and hence electric fields and these electric fields are disturbed by electric field waves. So, every material is heated by electric field waves. That is, every material is heated by light rays. Every living being realizes heat from radio waves, in particular from cell phone waves. If wavelength of light rays is decreased, then heat is increased in materials.

5. BOUNDS FOR WAVELENGTHS

A finite real constant s is called a lower bound of a non empty set A of real numbers, if $s \le t$, for every real number t in the set A. A finite real constant s is called an upper bound of a non empty set A of real numbers, if $t \le s$, for every real number t in the set A. The number $\Lambda = 1.063022744 \times 10^{-3}$ is an upper bound for the set of all wavelengths of all electric field waves, because wavelength of every electric field wave is less than or equal to this number.

The same number $\Lambda = 1.063022744 \times 10^{-3}$ is a lower bound for the set of all wavelengths of all magnetic field waves, because wavelength of every magnetic field wave is greater than or equal to this number. Is there any positive lower bound for the set of all wavelengths of all electric field waves? Is there any finite upper bound for the set of all wavelengths of all magnetic field waves? The answers for both questions are "yes". Let us find these bounds in this section.

For this purpose, let us recall some facts from the article [18] for black body radiations, which were obtained from the Planck's distribution. If $\lambda T \ll \frac{hc}{k}$, then $1.70356615852494 \times 10^{-3} \leq \lambda T$. It happens that every material having temperature more than 40541397546 Kelvin begins to become energy. So, for every wavelength λ of any ray radiated from a black body, the following relation should be true: $\lambda \geq \frac{1.70356615852494 \times 10^{-3}}{40541397546}$. This means that every wavelength λ of any electric field wave should satisfy the following relation: $\lambda \geq 4.202041028783 \times 10^{-20}$. This means that there is no electric field wave with wavelength less than $4.202041028783 \times 10^{-20}$ m. So, this number $4.202041028783 \times 10^{-20}$ is a lower bound for the set of all wavelengths of all electric field waves (Figure 1). Now, let us find a finite upper bound for the set of all wavelengths of all magnetic field waves.

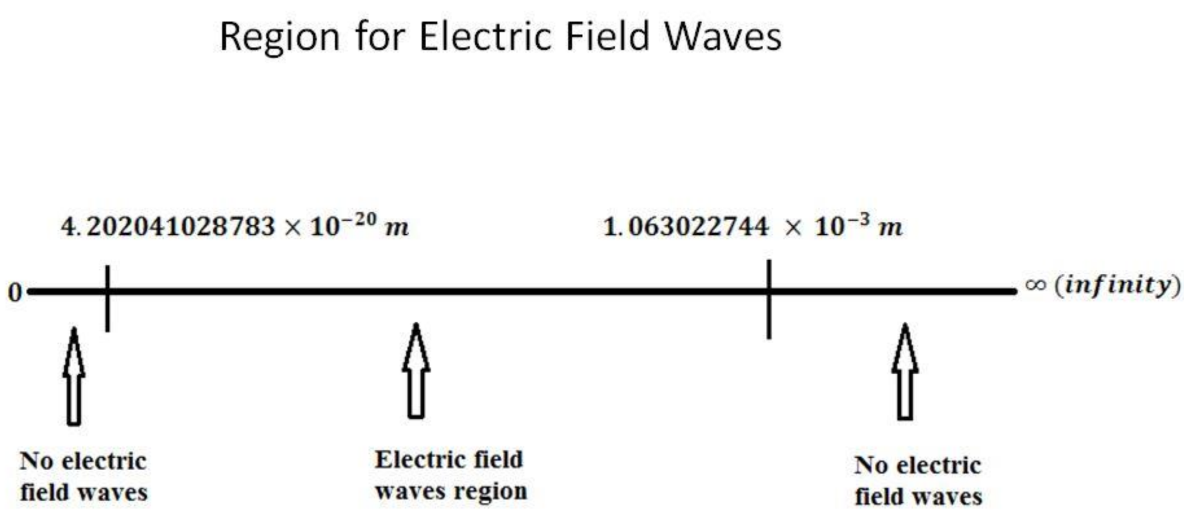

Figure 1. Bounds for electric field waves.

Let $\lambda_e = 4.202041028783 \times 10^{-20}$. Let $\lambda_m = \frac{1.129996 \times 10^{-6}}{\lambda_e}$. If λ_e represents wavelength of an electric field wave, then λ_m represents wavelength of a magnetic field wave such that both waves have same energy as well as both of them have same temperature, theoretically.

Since there is no electric field wave with wavelength less than this number λ_e, there is no magnetic field wave with wavelength greater than this number λ_m.

That is, $\lambda_m = 2.6891598446 \times 10^{13}$ is an upper bound for all wavelengths of all magnetic field waves (Figure 2). Let us now conclude finally in connection with bounds for electromagnetic waves. If λ_1 represents wavelength of an electric field wave, and λ_2 represents wavelength of a magnetic field wave, then (Figure 3):

$$4.202041028783 \times 10^{-20}\, m \leq \lambda_1 \leq 1.063022744 \times 10^{-3}\, m$$

and

$$1.063022744 \times 10^{-3}\, m \le \lambda_2 \le 2.6891598446 \times 10^{13}\, m.$$

If λ represents wavelength of an electromagnetic wave then:

$$4.202041028783 \times 10^{-20}\, m \le \lambda \le 2.6891598446 \times 10^{13}\, m.$$

Thus, $4.202041028783 \times 10^{-20}\, m$ is a lower bound for the set of all electromagnetic waves and $2.6891598446 \times 10^{13}\, m$ is an upper bound for the set of all electromagnetic waves (Figure 4).

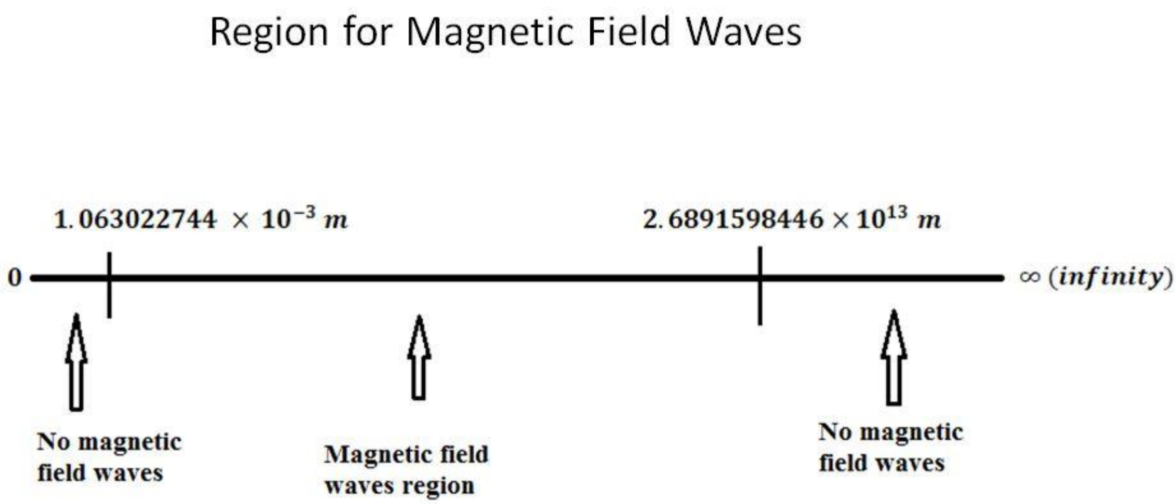

Figure 2. Bounds for magnetic field waves.

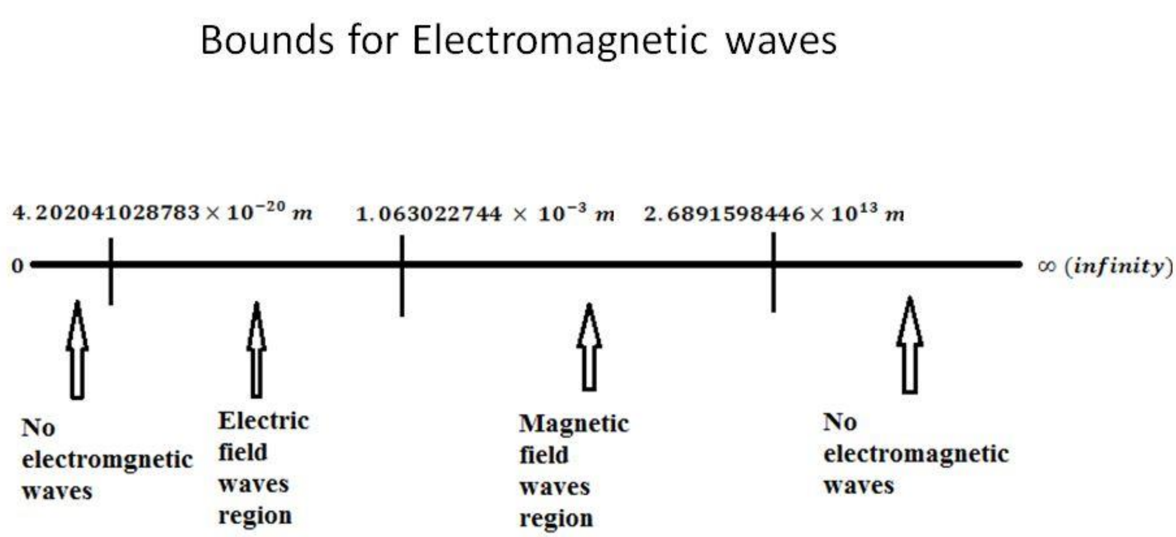

Figure 3. Bounds for electric field waves and magnetic field waves.

It should be pointed out that the most important bound known is Chandrasekhar's limit [3] meant for maximum size for black holes and for a limit for a big bang [14]. A different approach for a derivation of Chandrasekhar's limit may be seen in the book "Planets and electromagnetic waves" of the authors.

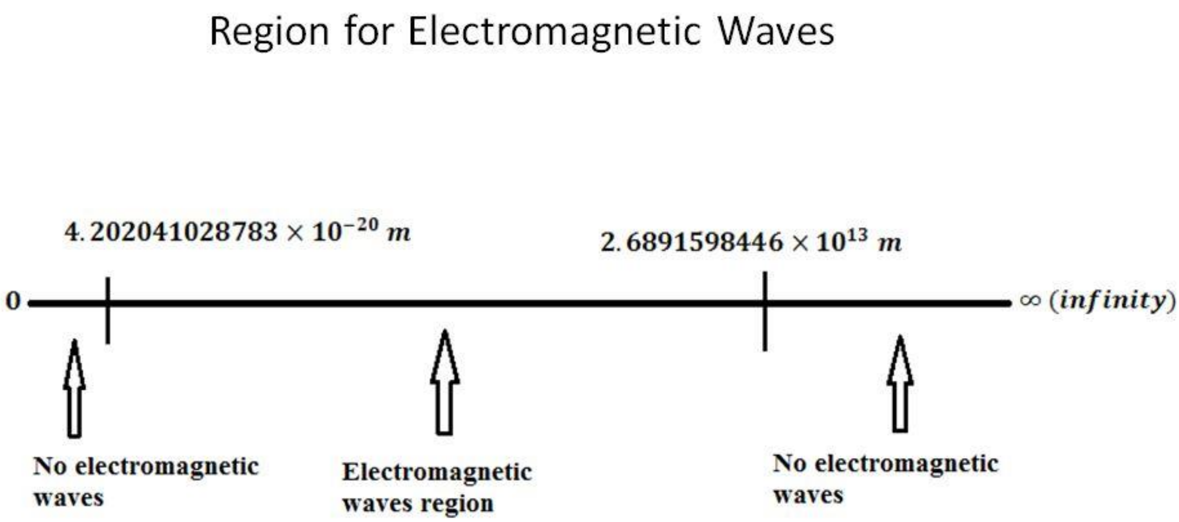

Figure 4. Bounds for electromagnetic waves.

6. ACCURACY

All constants in physics are found and refined again and again [16, 24]. One may find a number of articles [2, 6, 7, 15, 16, 22, 25] to find and refine the physical constants $h, c, k, b,$ and temperature of cosmic microwave background, which are used in the present article. Based on accuracy of these constants and on accuracy on evaluation of the expressions in the present article, it is possible to improve the bounds for electric field waves, magnetic field waves, and electromagnetic waves, and to improve formulas for temperatures of electromagnetic waves. Integrals in connection with $u(\lambda)$ have been found from zero to infinity, when deriving interesting displacement laws in Section 2. But the function values $u(\lambda)$ are meaningful only in the restricted interval

$$4.202041028783 \times 10^{-20}\, m \leq \lambda \leq 1.063022744 \times 10^{-3}\, m.$$

So, the integrals should be restricted only to this interval while evaluating them. This means that the interesting displacement laws based on integrals, derived in Section 2, are approximate displacement laws, and they are not exact displacement laws. However, the Wien's displacement law is based only on derivatives, not on integrals. So, the Wien's displacement law is an exact displacement law. Again, a reason is found for the suitability of the mode value of Planck's distribution for representation of the entire Planck's distribution. One more thing should be stated regarding bounds of the wavelengths of electric field waves. Depending on the physical constants $h, c, k, b,$ and temperature of cosmic microwave background, there may be a change in the accuracy of the one bound $1.063022744 \times 10^{-3}\ m$. The other bound $4.202041028783 \times 10^{-20}\ m$ is just "a" lower bound, and it is not "the" lower bound for all wavelengths of all electric field waves. There is always a chance to increase the value of this bound, when it is possible to find exact "the" upper bound for all wavelengths of all gamma rays. Similarly, there is a chance to decrease the upper bound value $2.6891598446 \times 10^{13}\ m$ for all wavelengths of all magnetic field waves.

Thus, there are chances to increase the value $4.202041028783 \times 10^{-20}\ m$ and to decrease the value $2.6891598446 \times 10^{13}\ m$. The theory of the present article is not about the accuracy, but about existence of bounds for wavelengths of electric field waves, bounds for wavelengths of magnetic field waves, and bounds for wavelengths of electromagnetic waves, and about existence of the best interesting Wien's displacement law and Planck's constant for magnetic field waves to define temperature of electromagnetic waves. Analysis on accuracy also favours "the" definition of temperature of electromagnetic waves through the Wien's displacement law.

The following two short tables provide only approximate wavelengths for easy evaluation purpose. The first one provides temperature for electric field waves (Table 1), and the second one provides temperature for magnetic field waves.

Table 1. Temperatures for electric field waves.

Types of rays	Wavelength λ (in metre)	Temperature $T_\lambda = \frac{2.8978 \times 10^{-3}}{\lambda}$ (in Kelvin)
Gamma ray	10^{-12}	2.8978×10^{9}
X-ray	10^{-10}	2.8978×10^{7}
Violet ray	10^{-8}	2.8978×10^{6}
Far infrared ray	10^{-3}	2.8978

Table 2. Temperatures for magnetic field waves.

Types of waves	Wavelength λ (in metre)	Temperature $T_\lambda = 2.564434 \times 10^{3} \times \lambda$ (in Kelvin)
5G cell phone waves	10^{-1}	2.564434×10^{2}
2G cell phone waves	10^{1}	2.564434×10^{4}
Radio waves	10^{2}	2.564434×10^{5}
Gravitational waves	10^{3}	2.564434×10^{6}

These tables illustrate the followings. Temperatures of electric field waves increase when wavelengths decrease, and temperatures of magnetic field waves increase when wavelengths increase. Wavelengths are decreased or equivalently frequencies are increased, when generations of cell phones are improved, and this idea is good because temperatures are decreased. Ultraviolet rays are better than infrared rays in killing germs, because temperatures for ultraviolet rays are greater than temperatures of infrared rays. Infrared rays are better than ultraviolet rays in fibre communications [22], because of better total reflections and because temperatures of infrared rays are lesser than temperatures of ultraviolet rays. If there are fluid flows in a body, then it may cause magnetic fields and thereby the body may be heated by magnetic field waves. If germs do not have fluid flows then magnetic field waves may not kill germs. But germs do contain electrons and electric fields so that electric field waves heat them.

So, germs may be killed by ultraviolet rays.

7. CONCLUSIONS

There is a formula for temperature of electric field waves, by using the Wien's displacement law. There is a formula for temperature for magnetic field waves by using the Planck's equation for magnetic field waves and by using the formula for temperature of electric field waves. Thus there are formulas for measuring temperature of all electromagnetic fields, because every electromagnetic field is either an electric field wave, or exclusively, a magnetic field wave. There is a constant $\Lambda = 1.063022744 \times 10^{-3}\, m$, such that it is an upper bound for the set of all wavelengths of all electric field waves and it is a lower bound for the set of all wavelengths of all magnetic field waves. The number $4.202041028783 \times 10^{-20}\, m$ is a lower bound for the set of all wavelengths of all electric field waves and it is a lower bound for the set of all wavelengths of all electromagnetic waves. The number $2.6891598446 \times 10^{13}\, m$ is an upper bound for the set of all wavelengths of all magnetic field waves and it is an upper bound for the set of all electromagnetic waves. Temperature of electric field waves increases with decrease in wavelength. Temperature of magnetic field waves increases with increase in wavelength. Entire theory is applicable for all continuous energy waves which have the speed c in vacuum. Entire theory is applicable only for electromagnetic waves for which wavelengths can be found experimentally based on a correct principle.

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