Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 183 (2023) 121-132 EISSN 2392-2192

A Dark Energy Model in Kaluza-Klein Framework with a Time Dependent Cosmological Constant

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ABSTRACT

We formulate a cosmological model under the Kaluza-Klein framework, in flat space, to determine the nature of time dependence of some cosmological parameters. Field equations are solved with the help of suitable ansatzes for the equation of state (EoS) parameter and the cosmological constant. To determine the arbitrary constants associated with the model, the currently accepted values of some measurable parameters are used. To examine the effect of dark energy upon the cosmic evolution, we have plotted the scale factor and its time derivative (expansion rate) with respect to a function of the cosmological constant which represents dark energy. Time evolution of different cosmological quantities is shown graphically and these graphs are consistent with the results of some recent investigations. It is observed that, the gravitational constant increases with time. The cosmological constant is positive and it decreases with time at a gradually decreasing rate. The EoS parameter is positive in the early universe when it decreases steeply, becoming negative eventually with a much slower variation thereafter. Results of this study are in agreement with the findings based on recent observational data.

***Keywords*:** Kaluza-Klein space-time, Dark energy, Accelerated expansion, Cosmological parameters, Gravitational constant, Cosmological constant

(Received 08 July 2023; Accepted 30 July 2023; Date of Publication 01 August 2023)

1. INTRODUCTION

On large scales, the universe is highly homogeneous and isotropic, based on the observations on large scale structures and Cosmic Microwave Background Radiation (CMBR) [1, 2]. Using observational results, it has been established beyond doubt that the universe is expanding with acceleration at the present time [3-7]. This acceleration is believed to be caused by energy of an exotic form which generates negative pressure. This is known as dark energy (DE) whose nature is still a mystery, and no explanation has been found to be satisfactory about its origin [8]. A parameter, named cosmological constant (Λ), which was a part of the General Theory of Relativity (GTR) proposed by Einstein, is used in many calculations as a representative of DE. It has explained several experimental observations satisfactorily although it has its limitations [9]. The equation of state (EoS) parameter (ω) (ratio of pressure to density of the cosmic fluid), which characterizes DE conventionally, should no longer be treated as a constant. Due to insufficient observational data to estimate how ω evolves with time, the EoS parameter has been considered to be time independent. Its values are +1, 1/3, 0, -1, respectively, for a universe filled with stiff fluid, radiation, pressure-less dust, vacuum [9]. The EoS parameter varies with time or redshift [10]. Several models have been constructed in recent times regarding the time evolution of ω [11-13].

Initially it was believed that the cosmic expansion is controlled solely by gravitational attraction among celestial bodies, which would only cause decelerated expansion. The negative value of the deceleration parameter (q), obtained from observations, confirmed accelerated expansion of the universe. To unify electro-magnetic force with gravitational force, Kaluza and Klein made an attempt in the first half of the last century, giving birth to what is known as Kaluza-Klein (KK) theory. A fifth dimension was introduced for coupling these two forces in KK approach. It was shown by Kaluza that a five dimensional formulation of GTR consists of the four dimensional conventional GTR along with the electromagnetic field which is consistent with Maxwell's electromagnetism [14]. Kaluza extended GTR to five-dimensions, and Klein achieved the compactification of the fifth dimension [15]. In their five dimensional models, Chodos and Detwelier demonstrated that cosmic evolution causes the contraction of the extra dimension [16]. Based on investigations by Guth, Alvarez and Gravela, a large entropy is produced due to an extra dimension which is capable of providing solutions to the flatness and horizon problems without having to take recourse to the theories of inflation [17, 18]. Thus, some of the problematic issues of Big Bang Cosmology and other areas of physics has been successfully addressed by a model in five-dimensions under the framework of KK theory.

Scientific literature provides a number of works where any one or both of the cosmological parameters, Λ and G, are considered to be variables. Time variation of various cosmological parameters have been found from the FRW models formulated by Ray et al. using the ansazes of $\Lambda \sim H^2$ and $\Lambda \sim \ddot{a}/a$ [19]. In the study by Khadekar et al and Ozel et al, in KK framework, G is a constant but Λ is a variable quantity [20, 21]. Considering both of Λ and G to be variables, Oli and the group of Sharif and Khanum have constructed their models in KK framework [22, 23]. A model in KK space-time with $\Lambda \sim \ddot{a}/a$ has recently been constructed by Mukhopadhyay et al where the time variations of both Λ and G have been determined [24].

The objective of the present study is to determine the time dependence of various cosmological parameters by constructing a model under the Kaluza-Klein framework, for zero spatial curvature. This model involves the dynamical Λ term which is said to represent the effect of dark energy. Empirical expressions for Λ and ω have been used to solve the field equations.

The arbitrary constants associated with these expressions, have been determined using the presently accepted values of some measurable cosmological parameters. Time evolution of various parameters, obtained from this model, has been shown graphically. The findings of the present study are consistent with those of many other recent studies carried out in different theoretical frameworks.

2. FIELD EQUATIONS

Kaluza-Klein space-time metric is expressed as,

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 + (1-kr^2)d\psi^2\right] \quad (1)$$

In equation (1), $d\Omega^2 = d\theta^2 + sin^2\theta\, d\phi^2$. $a(t)$ is the scale factor. k denotes spatial curvature whose values are -1, 0 and 1 respectively for spatially open, flat and closed universe.

Einstein's field equations are given below.

$$R_{\alpha\beta} - \frac{1}{2}R\, g_{\alpha\beta} = 8\pi G\, T_{\alpha\beta} - \Lambda\, g_{\alpha\beta} \quad (2)$$

The symbol Λ, in equation (2), denotes the time-variable cosmological constant.

The energy-momentum tensor, for a perfect fluid (which is considered to be occupying the entire universe), is expressed as,

$$T_{\alpha\beta} = (p+\rho)u_\alpha u_\beta - p g_{\alpha\beta} \quad (3)$$

where $\alpha, \beta = 0,1,2,3,4$. Here u_α satisfies the relation $u^\lambda u_\lambda = 1$. The symbols ρ and p stand for the energy density and pressure, respectively, of the perfect fluid occupying the universe.

Using the three equations above, we get the relations expressed by equations (4) and (5), using which, the time dependence of various cosmological parameters can be examined.

$$8\pi G\rho = 6\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \Lambda \quad (4)$$

$$8\pi G p = -3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \Lambda \quad (5)$$

For a spatially flat universe (where the curvature parameter $k = 0$), equations (4) and (5) get transformed into the following equations.

$$8\pi G\rho + \Lambda = 6\left(\frac{\dot{a}}{a}\right)^2 = 6H^2 \quad (6)$$

$$8\pi G p - \Lambda = -3\dot{H} - 6H^2 \quad (7)$$

In obtaining equations (6) and (7), from (4) and (5) respectively, one has to use the Hubble parameter expression, i.e., $H = \frac{\dot{a}}{a}$.

The continuity equation is given by,

$$\dot{\rho} + 4H(p + \rho) = 0 \quad (8)$$

The relation between pressure and density, for a barotropic equation of state, is given by,

$$p = \omega\rho \quad (9)$$

In equation (9), ω is designated as the EoS (equation of state) parameter.
Combining equations (7) and (9) we get,

$$8\pi G\omega\rho = \Lambda - 3\dot{H} - 6H^2 \quad (10)$$

Based on a combination of equations (8) and (9) we get equation (11), as given below.

$$\dot{\rho} + 4H\rho(\omega + 1) = 0 \quad (11)$$

Based on a combination of equations (6) and (10) we get equation (12), as given below.

$$\dot{H} = \frac{(\omega+1)(\Lambda-6H^2)}{3} \quad (12)$$

3. SOLUTION OF FIELD EQUATIONS

In this part of our study, we propose an empirical mathematical model involving two ansatzes, one relating the EoS parameter (ω) with time and the other relating the cosmological constant (Λ) with the Hubble parameter (H). These ansatzes are given by,

$$\omega + 1 = At^l \quad (13)$$

$$\Lambda - 6H^2 = 3BH^r \quad (14)$$

where A, B, l and r are arbitrary constants.
Using equations (13) and (14) in equation (12), we obtain

$$\dot{H} = ABt^lH^r \quad (15)$$

which on integration yields,

$$H = \frac{\dot{a}}{a} = \left[\frac{AB(1-r)}{l+1}\, t^{(l+1)} + C_1(1-r)\right]^{\frac{1}{1-r}} \quad (16)$$

where C_1 is the constant of integration. Equation (16) is a differential equation whose analytical solution can be obtained for $C_1 = 0$. This solution is given below.

$$a = C_2 exp\left[\left\{\frac{AB(1-r)}{l+1}\right\}^{\left(\frac{1}{1-r}\right)}\left(\frac{1-r}{2+l-r}\right)t^{\left(\frac{2+l-r}{1-r}\right)}\right] \tag{17}$$

where C_2 is an integration constant.

The deceleration parameter (q), derived from equation (17), is given by,

$$q = -\frac{\dot{H}}{H^2} - 1 = -(AB)^{\left(\frac{1}{r-1}\right)}\left(\frac{1-r}{l+1}\right)^{\left(\frac{r-2}{1-r}\right)} t^{\left(\frac{l-r+2}{r-1}\right)} - 1 \tag{18}$$

Using equations (13) and (16) in equation (11), for $C_1 = 0$, and integrating, we obtain the following expression for the energy density (ρ).

$$\rho = C_3 exp\left[-4A\left\{\frac{AB(1-r)}{l+1}\right\}^{\left(\frac{1}{1-r}\right)}\left\{\frac{1-r}{(l+1)(2-r)}\right\} t^{\frac{(l+1)(2-r)}{1-r}}\right] \tag{19}$$

where C_3 is the constant of integration.

Using equations (14), (16) and (19) in equation (6), for $C_1 = 0$, we obtain the following expression for the gravitational constant (G).

$$G = -\frac{3B}{8\pi C_3}\left[\frac{AB(1-r)}{l+1}t^{(l+1)}\right]^{\left(\frac{r}{1-r}\right)} exp\left[4A\,\frac{AB(1-r)}{l+1}^{\left(\frac{1}{1-r}\right)}\frac{1-r}{(l+1)(2-r)}t^{\frac{(l+1)(2-r)}{1-r}}\right] \tag{20}$$

Using equation (16) in (14), for $C_1 = 0$, we obtain the following expression for the cosmological constant (Λ).

$$\Lambda = 3B\left[\frac{AB(1-r)}{l+1}t^{(l+1)}\right]^{\left(\frac{r}{1-r}\right)} + 6\left[\frac{AB(1-r)}{l+1}t^{(l+1)}\right]^{\left(\frac{2}{1-r}\right)} \tag{21}$$

4. RESULTS AND DISCUSSION

It is necessary to know the values (or the ranges of variation) of the constant parameters connected with this model, to determine the time evolution of some cosmological quantities (a, H, q, ρ, G, Λ, ω, $\dot{G}/G$).

The model parameters (or the ranges of their variation) can be estimated from the following requirements: 1) $a > 0$, 2) $H > 0$, 3) the values of H_0, q_0, ρ_0, G_0, as predicted by this model, must be sufficiently close to the values obtained from astrophysical observations, 4) the evolution of the deceleration parameter, as predicted by this model, should manifest a signature flip from positive to negative, indicating (as per observational data) a change from decelerated expansion to accelerated expansion [3-7].

Based on these requirements, we have obtained the following set of values for the model parameters.

$$l = -0.999, r = 1.002, A = 1.943 \times 10^{17}, B = -2.510 \times 10^{-18},$$
$$C_2 = 0.143, C_3 = 1.579 \times 10^{-28}.$$

We have chosen $a_0 = 1$ in this study.

The currently accepted values (in SI units) of some cosmological parameters, used for this purpose, are given below.

$$t_0 = 4.34 \times 10^{17}\,sec, \quad \rho_0 = 9.90 \times 10^{-27} Kg/m^3, \; q_0 = -0.55,$$
$$G_0 = 6.67 \times 10^{-11} N\,m^2/\,Kg^2, H_0 = 2.39 \times 10^{-18}\,sec^{-1}.$$

Figure 1 depicts time evolution of the scale factor (a). It is found to increase monotonically with time, which is true for an expanding universe. Figure 2 shows the variation of Hubble parameter (H) as a function of time. It is observed that H decreases with time with a decreasing slope. This nature of time evolution is similar to the results of other recent investigations [24, 25].

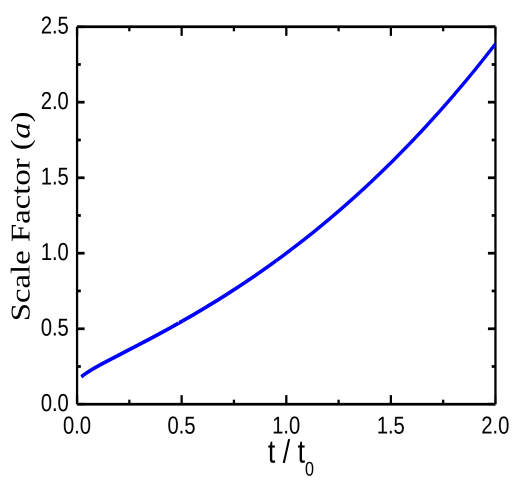

Figure 1. Variation of scale factor (a) as a function of time

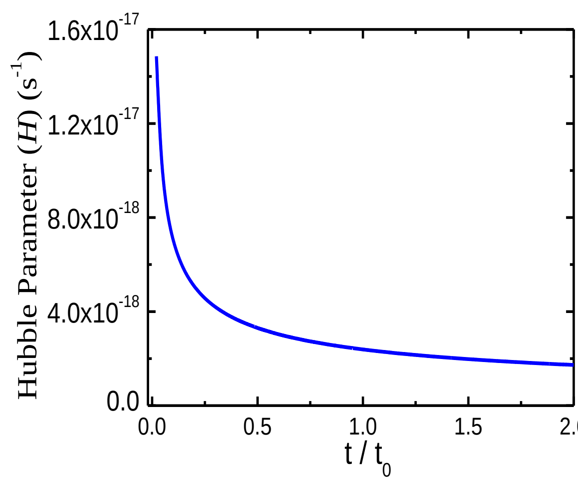

Figure 2. Variation of Hubble parameter (H) as a function of time

The variation of the deceleration parameter, with respect to time, is depicted by Figure 3. It exhibits a signature flip implying a change of phase from decelerated expansion to accelerated expansion, as obtained from observational data [3-7]. According to this figure, the change of phase took place nearly at $t = 0.23t_0$. Its present value is negative and it decreases with time at a gradually decreasing rate, having sufficient consistency with the results of many other studies [13, 25-30].

Time evolution of energy density (ρ) is depicted (in logarithmic scale) by Figure 4. It has a negative slope which becomes less negative with time. This behaviour is in agreement with the findings of many other investigations [12, 13, 24-31].

Time evolution of gravitational constant (G) is depicted by Figure 5. It is found to increase with time, which is consistent with the findings of other recent studies [23, 24, 28-31].

Figure 6 depicts the time evolution of the equation of state (EoS) parameter ω. It is found to decrease steeply with time in the early universe (when it was positive) and it continues to decrease at a decreasing rate. It changes sign from positive to negative nearly at $t = 0.5t_0$. It is in the quintessence ($\omega > -1$) region for the entire span of time depicted in the graph and it approaches -1 (vacuum fluid dominated universe) as $t \to \infty$, in accordance with equation (13) where l is a negative quantity. This behaviour of ω is in agreement with the results of some other studies [13, 27, 32]. Its present value (i.e., ω_0) is -0.548.

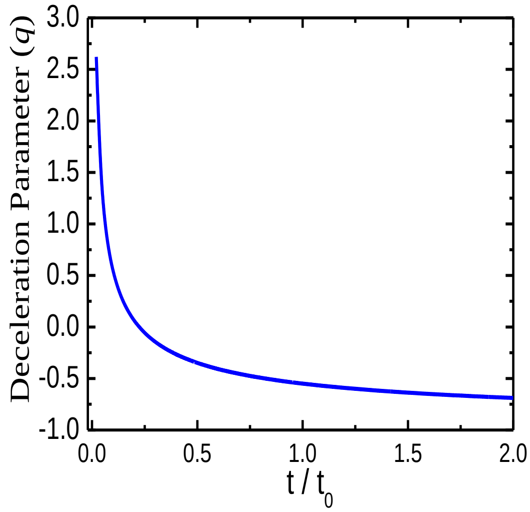

Figure 3. Variation of deceleration parameter (q) as a function of time

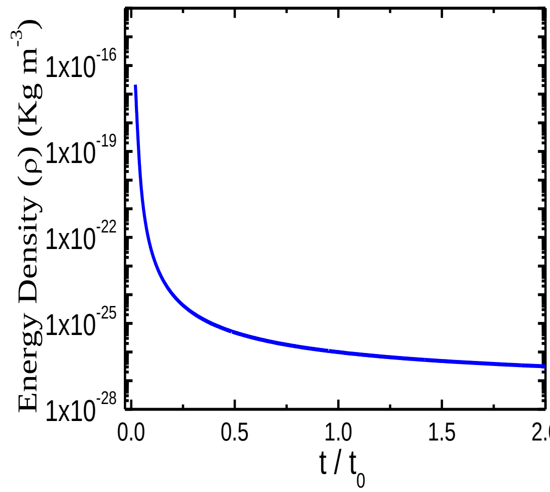

Figure 4. Variation of energy density (ρ) as a function of time

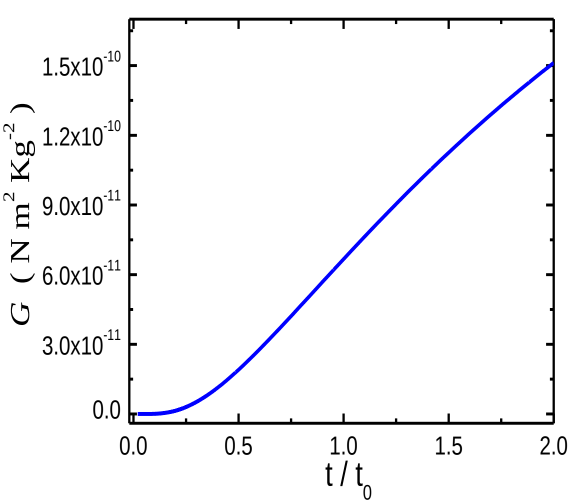

Figure 5. Variation of the dynamical G as a function of time

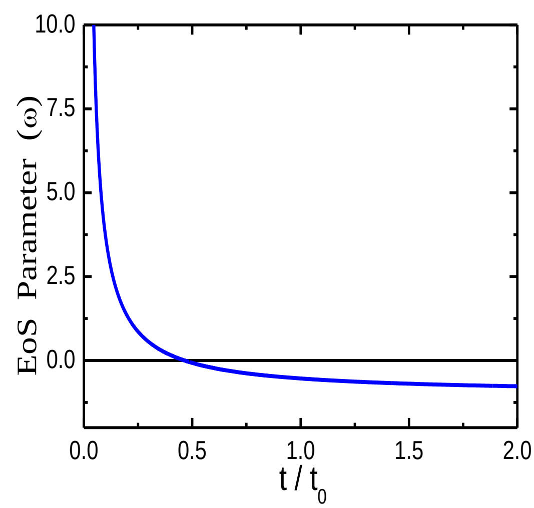

Figure 6. Variation of the EoS parameter (ω) as a function of time

Time dependence of the dynamical cosmological term (Λ) is shown by Figure 7. It is positive and it decreases with time. It decreases steeply in the early universe, followed by a phase of much slower change. Its behaviour is consistent with the results of studies based on various other models [13, 23, 24, 28, 30]. It is consistent with the variation, $\Lambda \propto t^{-2}$, as mentioned by several authors [33]. According to cosmological observations, we have a small and positive Λ at present epoch [3, 34]. Thus, the behaviour of Λ, as per our model, is consistent with recent observations.

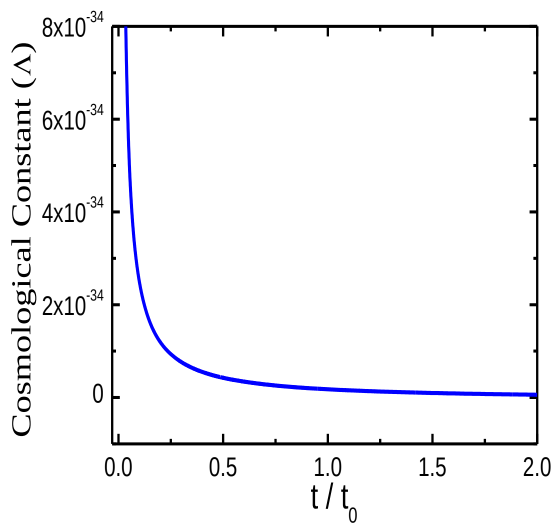

Figure 7. Variation of the dynamical Λ as a function of time

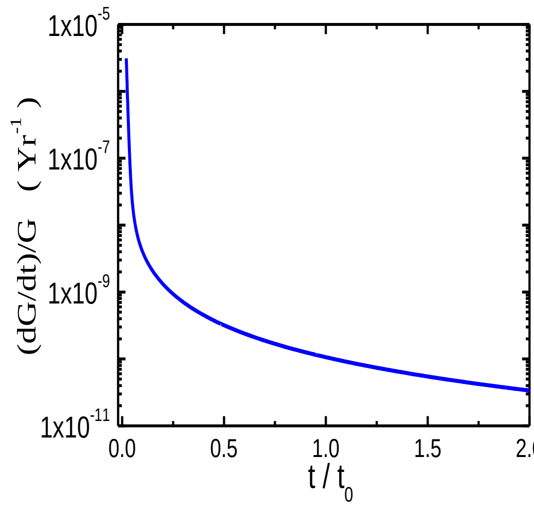

Figure 8. Variation of $\dot{G}/G$ as a function of time

Figure 8 shows the variation of $\dot{G}/G$ as a function of time. For the sake of clarity, it has been plotted in the logarithmic scale. It is positive, as expected from the variation of G in Figure 7. It decreases with time at a gradually decreasing rate, which is consistent with the findings of one of our previous studies in Brans-Dicke framework [31]. Its present value is found to be $1.075 \times 10^{-10}\ Year^{-1}$ which is consistent with the values obtained from various observations [19].

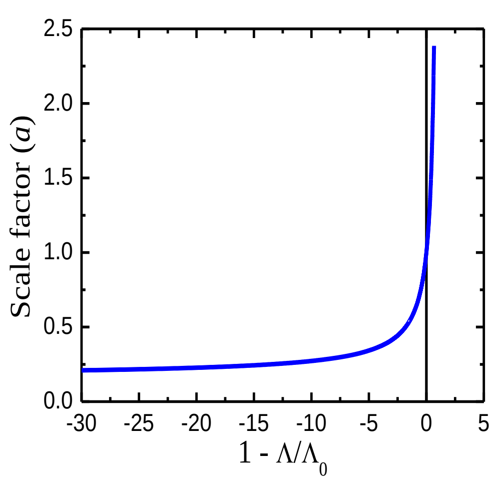

Figure 9. Variation of scale factor (a) with respect to $1 - \Lambda/\Lambda_0$

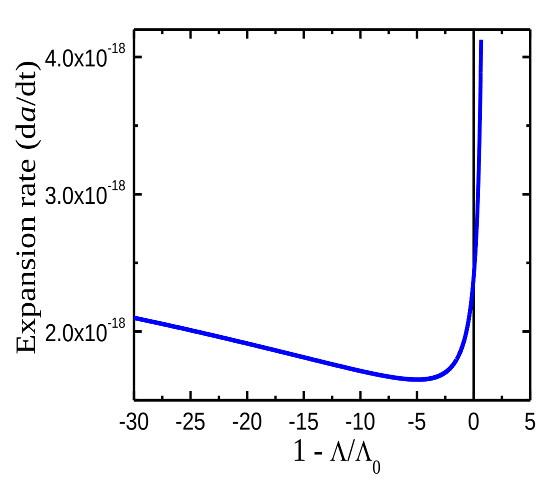

Figure 10. Variation of cosmic expansion rate (da/dt) with respect to $1 - \Lambda/\Lambda_0$

Figure 9 shows the variation scale factor (a) with respect to $(1 - \Lambda/\Lambda_0)$. Λ is known to decrease with time (Figure 7). As per Figure 9, when Λ reaches a value close to $3.5\Lambda_0$, the time evolution of the scale factor is much more rapid compared to that of Λ. Figure 10 shows a plot of cosmic expansion rate (da/dt) with respect to $(1 - \Lambda/\Lambda_0)$. da/dt starts rising very rapidly at around $\Lambda = 3.5\Lambda_0$. These observations may be connected to some phenomena involving dark energy (represented by Λ) which causes accelerated expansion of the universe.

5. CONCLUDING REMARKS

In the present study, we have constructed a cosmological model in the framework of Kaluza-Klein space-time (for zero spatial curvature), where, the parameters ω, G and Λ have been regarded as time dependent quantities. To validate this model, we have used the currently accepted values of some measurable parameters (H, q, ρ, G) at the present time (i.e., $t = t_0$). Based on this model, we have determined the time dependence of some cosmological quantities and depicted them graphically. It is observed that, G increases with time, while Λ and ω decrease as time goes on. An important feature of the present study is that we have obtained the time evolution characteristics of various parameters (a, H, q, ρ, G, ω, Λ, $\dot{G}/G$) from a single model whose formulation is mathematically simpler than many other models whose findings are in agreement with those of this one. The results of this study are consistent with the observed features of the expanding universe. As an extension of this work, one may use a set of ansatzes (different from those expressed by eqns. 13 and 14), and compare the results obtained thereby with those of the present study.

Acknowledgement

The content of the present article is based on a part of research undertaken by Asmita Das, Anwesha Dey, Debolina Biswas and Sudipto Saha Roy, under the supervision of Dr. Sudipto Roy, for a dissertation paper in the final year of the M.Sc. (Physics) course at St. Xavier's College, Kolkata, West Bengal, India, during the period from 2020 to 2022. Dr. Sudipto Roy and his co-authors are thankful to their institution for providing them with the opportunity to carry out this research.

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