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Total Dominating Sets in Wireless Sensor Networks with Application of Dominating Sets

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ABSTRACT

In this paper, A set $S \subseteq V$ is a dominating set of a graph G if for every $v \in V$, either $v \in S$ or $v \in N(u)$ for some vertex $u \in S$ where N(u) represents neighbourhood of u. The minimum cardinality of a dominating set in a graph G is called the domination number of G, and is denoted by $\gamma(G)$. A subset $S \subseteq V$ in a graph $G = (V, E)$ is a total [1, 2]-set if, for every vertex $v \in V, 1 \leq |N(v) \cap S| \leq 2$. The minimum cardinality of a total [1, 2]-set of G is called the total [1, 2] domination number, denoted by $\gamma_{t[1,2]}(G)$. The hub of this article is a search of the behaviour of Application of dominating sets, total dominating sets in wireless sensor networks.

***Keywords*:** Dominating set, Minimum cardinality of a total, Application of dominating sets, total dominating sets in wireless sensor networks, Dominating set of a graph

AMS Subject Classification: 05C69, 05C76

1. INTRODUCTION

The study of dominating sets and dominating numbers are very well studied research in graph theory. Research models based on it have several applications in computer

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communication networks, Radio stations, Radar stations, Molecular physics and chemistry, Biological sciences, Engineering and in other numerous areas. A Wireless Sensor Network (WSN) is also one of the application graph models. A WSN is an ad-hoc network of sensors where sensors sense and process data information.

The ability of completing all the tasks under emerging WSN applications is called surviving capability. It endures the failures to achieve resilience and make energy management to maximize the network lifetime. In the network of sensors every region has multiple nodes.

So we can cover a region by making one of the nodes in the region awake while making the rest of the nodes in a sleep state. In the sleep state, a sensor does not do radio transmission or sensing.

The radio transmission consumes considerable power. By change the state of the nodes from sleep to awake and vice versa to reduce power consumption and to improve the lifetime of WSN Various types of conservation algorithms introduced to enhance the surviving capabilities in order to assure the coverage of a WSN and also to increase its lifetime. A scheduling procedure of sleeping and awaking- of nodes that is a node is kept in sleeping state while other awaking nodes distribute or gather the information to a dominating set. Partition method in a unit disk graph is used to produce that maximum disjoint dominating sets.

A WSN have computers and processors. It can be represented by a graph $G = (V, E)$ where vertices are computers and edges are links between two computers. Each processor collects information from all processors to which it is directly connected. A processor can do this task if it has information to one of a small set of collecting processors that is dominating set.

Information should be communicated over a small distance path. We need a small set of processors which are close to all other processors. In such a situation we can construct set of processors available at either one or two-distances that is [1, 2] sets. So the use of [1, 2] dominating sets can send the information early than three or four distances sets.

A set $S \subseteq V$ is a dominating set of a graph G if for every $v \in V$, either $v \in S$ or $v \in N(u)$ for some vertex $u \in S$ where N(u) represents neighbourhood of u. The minimum cardinality of a dominating set in a graph G is called the domination number of G, and is denoted by $\gamma(G)$.
A dominating set S of G is called a total dominating set if for every vertex $v \in V(G)$, $N(v) \cap S \neq \emptyset$. The total domination number of G, denoted by $\gamma t(G)$, is the minimum cardinality of a total dominating set of G.

For any two integers j and k, a subset S V in a graph G = (V, E) is a (j, k)-set if, for every vertex $v \in V \setminus S$, $j \leq |N(v) \cap S| \leq k$ that is , each vertex $v \in V \setminus S$ is adjacent to at least j vertices, but not more than k vertices in D.

For special case, (1, 2)-dominating set in a graph $G = (V,E)$ is a set having property for every vertex $v \in V - S$, there is at least one vertex in S at a distance 1 from v and a second vertex in S at a distance at most 2 from v. The (1, 2) −domination number of G, denoted by $\gamma_{1,2}(G)$, is

The minimum cardinality of a (1, 2) −dominating set of G.

A subset $S \subseteq V$ in a graph G = (V, E) is a total [1, 2]-set if, for every vertex $v \in V$, $1 \leq |N(v) \cap S| \leq 2$. The minimum cardinality of a total [1, 2]-set of G is called the total [1, 2]-domination number, denoted by $\gamma_{t[1,2]}(G)$.

A Wireless Sensor Network (WSN) is a network where the sensors have sensing and processing ability. WSN are used in fire monitoring, intelligent transport systems, environmental monitoring, our daily life security etc.

2. PROBLEM DESCRIPTION

An important Application of Dominating sets can be found in realising limited energy of the sensors in wireless sensor network (WSN). By distributing the data gathering and sensing tasks to a dominating set of awake sensors while the other nodes are in a sleep mode. Energy of these sensors is conserved by applying a sleep-wake scheduling of nodes. Dominate partition problem in unit disk graphs can be used to maximize the number of disjoint dominating sets. Maximum disjoint dominating sets increase network lifetime. Local multiple search algorithms can improve the total lifetime of WSNs consisting of nodes with varying initial energy. Dominating set algorithm can be applied on multiple disjoint dominating sets with nodes having varying initial energy.

The nodes that make up WSN have sensing, data processing, and communication capabilities. Low-cost and low-power nodes of WSNs communicate short distances only. To balance this energy related issue of WSNs, the network designer has to solve without affecting the data routing and network function.

The limited energy of the used sensors in WSNs makes them failures to complete targeted tasks and fail to prolong the network life-time. The radio transmission in a WSN uses considerable power. In the sleep state, the sensors stop radio transmissions and environment sensing. Energy will be conserved If we change the state of the nodes from sleep to awake and vice versa by deploying sensors in the network in such a way that every region is covered by multiple nodes. Now it is possible cover a region by making one of the nodes in the region awake while making the rest of the nodes in a sleep state. For this purpose, Guha and Khuller first use the dominating set algorithm to control the sleep-wake schedule of the nodes in a WSN then Islam *et al.* Introduce an algorithm to producing maximum number of disjoint dominating sets called the domatic partition problem in unit disk graphs.

Consider WSN having nodes *A*, *B*, *C*, *D*, *E*, *F* and *G* in following figure. The initial energy of each of the nodes and also their neighbors are given. Let [*A*, *B*, *E*] and [*C*, *D*, *F*] be two disjoint dominating sets. In set [*A*, *B*, *E*], the node *E* has the minimum initial energy (0.021).

In set [*A*, *B*, *E*] the node *D* has the minimum initial energy (0.175).

Lifetime of [*A*, *B*, *E*]:

$$\frac{0.021}{0.05} = 0.42$$

Lifetime of [*C*, *D*, *F*]:

$$\frac{0.175}{0.05} = 3.50$$

Total lifetime: 0.42+3.5 = 3.92.

For another selection of disjoint dominating sets Lifetime of [*B*, *D*,*E*] , [A,C,F]:

$$\frac{0.021}{0.05} + \frac{0.775}{0.05} = 15.92$$

For this selection lifetime is improved.

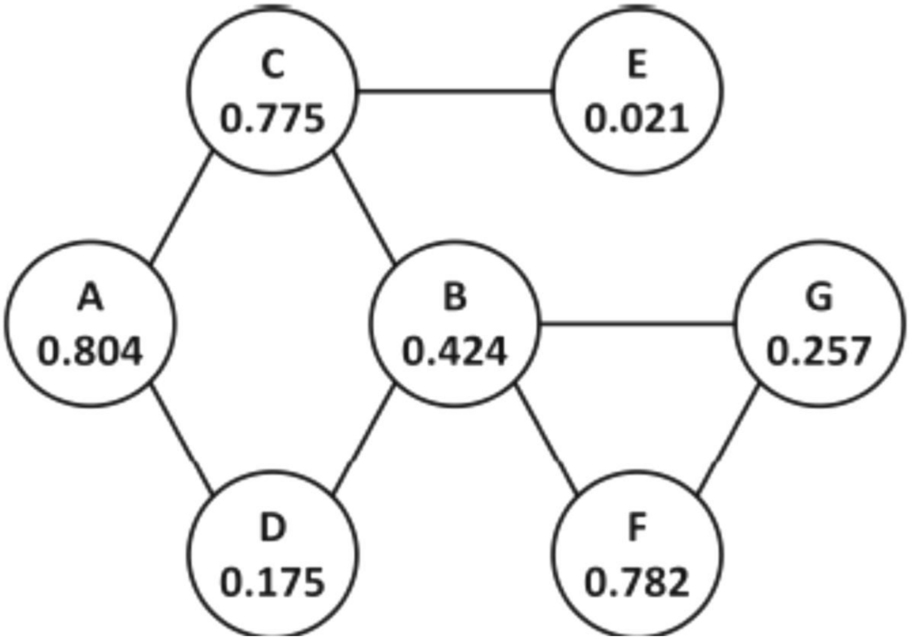

Figure 1. WSN Graph

Obtaining 1. Obtaining the initial feasible solutions of dominating sets from the given graph Construct Dominating set from a given arbitrary Graph G and formula to calculate its minimum dominating number:

Theorem 1: Let G = (V, E) be a graph with vertex set V = {v1; : : : ; vn} . Let G0 be a graph Obtained from G by adding a path Pi to any vertex vi of G. Then

$$\gamma(G0) = \begin{cases} \sum_{i=1}^{n} \lceil \frac{n_i}{3} \rceil & \text{if } 3|\, n_i \text{ or } 3|(n_i+1) \text{ for all } i \\ \sum_{i=1}^{n} \lfloor \frac{n_i}{3} \rfloor + \gamma(G) & \text{if } 3|\,(n_i-1) \text{ for all } i \end{cases}$$

Proof: Let n_i *denotes degree of a vertex then for any path graph* Pi

$\gamma(Pi) = \lceil \frac{n_i}{3} \rceil$.

Let $3|n_i$ *or* $3|(n_i+1)$. So $\gamma(G0) \geq \sum_{i=1}^{n} \lceil \frac{n_i}{3} \rceil$. Since vi = vi_1 is dominated by γ (G0)-set. Thus $\gamma(G0) = \sum_{i=1}^{n} \lceil \frac{n_i}{3} \rceil$.

Let $3|\,(n_i-1)$ then $\gamma\,(Pi - vi_1) = \frac{(n_i-1)}{3} = \lfloor \frac{n_i}{3} \rfloor$ and $\gamma(G0) \geq \sum_{i=1}^{n} \lceil \frac{n_i}{3} \rceil + \gamma(G)$.

Since vi = vi_1 is dominated by γ(G) set. Hence $\gamma(G0) = \sum_{i=1}^{n} \lceil \frac{n_i}{3} \rceil + \gamma(G)$.

Example: Consider Peterson Graph G = (V,E) with vertex set V = {1,2,3,4,5,6,7,8,9,10}. Here Set S = {1, 2, 9} is minimum dominating set so γ(G) = 3

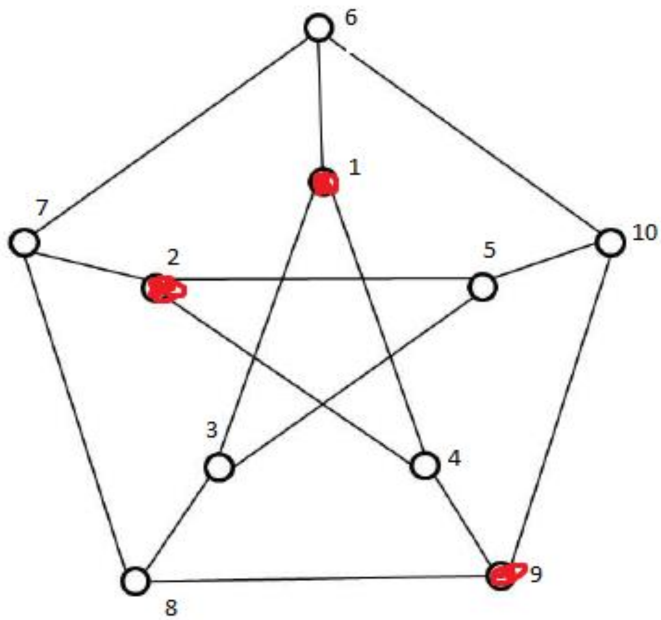

Figure 2. Peterson Graph

Obtain a graph G0 by adding a path P6 with vertices {11, 12, 13, and 14} at vertex 6 having 4 vertices i.e $n_6 = 4$.

Since 3 divide none of 4, 4+1 but 3 divides 4 - 1 = 3

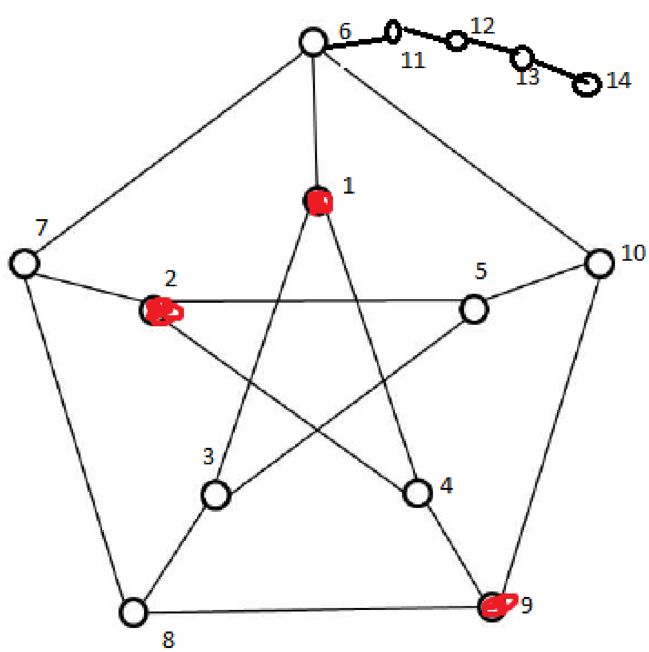

Figure 3. G_0 Graph by adding a path P6

$\gamma(P6) = \lceil \frac{4}{3} \rceil = 2$

So $\gamma(G0) = \gamma(P6) + \gamma(G) = 2+3 = 4$

Obtaining 2. Dominating sets from a given arbitrary graph G and formula to calculate its minimum dominating number:

Theorem 2: Let G = (V,E) be a graph with vertex set V = {v1; : : : ; vn}. Let G0 be a graph obtained from G by adding a path Pi to any vertex vi of G. Then

$$\gamma 2(G0) = \begin{cases} \sum_{i=1}^{n} \frac{n_i}{2} + \gamma(G) \text{ if } n_i \text{ is even for all } i \\ \sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil \quad \text{if } n_i \text{ is odd for all } i \end{cases}$$

Proof: If Pi be a path with n_i = 2k vertices then γ2(Pi - vi) $= \frac{n_i}{2}$, so that the vertex vi2 is in γ2(Pi - vi). Since the vertex vi1 = vi is dominated by vi2 and a vertex in γ (G)-set for all path Pi. Then $\gamma 2(G0) \geq \sum_{i=1}^{n} \frac{n_i}{2} + \gamma 2(G)$. Also the $U_{i=1}^{n}$ γ2(Pi - vi)-set U γ (G)-set 2- dominates G0. Thus $\gamma 2(G0) = \sum_{i=1}^{n} \frac{n_i}{2} + \gamma 2(G)$ if n_i is even for all i.
If Pi be a path with n_i = 2k+1 vertices, then γ2(Pi) $= \lceil \frac{n_i}{2} \rceil$ where the vertex vi must be in γ2(Pi) -set. Thus γ2(G0) $\sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil$ $\textit{if } n_i \textit{ is odd for all } i$

Example: Consider Generalised Peterson Graph P(3,1) = (V,E) with vertex set V = {1,2,3,4,5,6} it's 2- dominating number is γ2(G) = 3.

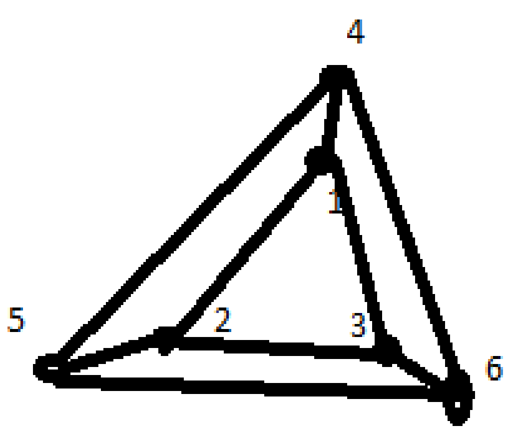

Figure 4. Generalised Peterson Graph P(3,1)

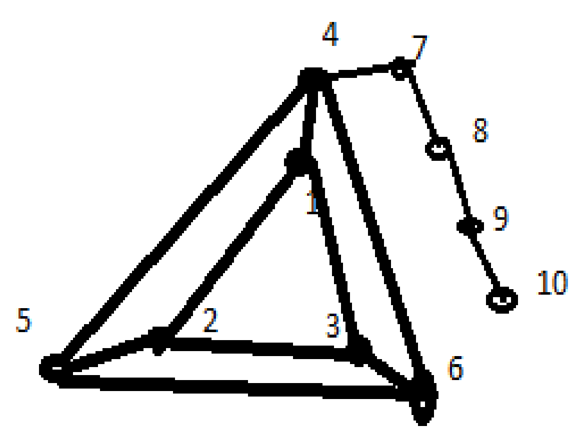

Figure 5. G_0 Graph

Obtain a graph G0 by adding a path P4 with vertices {7, 8, 9, and 10} at vertex 4 having 4 vertices i.e $n_4 = 4$ *is even*

Then $\gamma 2(G0) = \frac{n_4}{2} + \gamma(G) = 2+3 = 5$

3. TOTAL DOMINATING SET

Theorem 3: Let G = (V,E) be a graph without isolated vertex. Let V0 = {v1; : : : ; vn}. Let G0 be a graph obtained from G by adding a path Pi to any vertex vi of G. Then

$$\gamma t(G0) = \begin{cases} \sum_{i=1}^{n} \lfloor \frac{\boldsymbol{n_i}}{\boldsymbol{2}} \rfloor + \gamma t(G) if\ \boldsymbol{n_i} = \boldsymbol{4k_i} + 1\ for\ all\ i \\ \sum_{i=1}^{n} \lceil \frac{\boldsymbol{n_i}}{\boldsymbol{2}} \rceil \quad \boldsymbol{n_i} \neq \boldsymbol{4k_i} + 1\ for\ all\ i \end{cases}$$

Proof: Suppose that Pi is a path with ni vertices. Let $n_i = 4k_i + 1$. Since for any path we it's γt(Pi-vi) = 2*k_i= $\lfloor \frac{n_i}{2} \rfloor$. The graph G is totally dominated by γt(G).

So γt(G0) = $\sum_{i=1}^{n} \lfloor \frac{n_i}{2} \rfloor$ + γt(G) .

Let ni = 4ki. Then γt(Pi) = $2k_i$ where where the vertex vi = vi1 is totally dominated by γt(Pi)-set. Hence γt(G0) ≥ $\sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil$. Also these number vertices are needed for totally dominating G0. So γt(G0) = $\sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil$

Let ni = 4ki + 2 . By using the fact γt(Pi-{vi1,vi2}) = $2k_i$. If we put vi = vi1 in γt(G)- set. Then γt(G0) = $\sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil$. Let ni = 4ki + 3 then γt(Pi) = $2k_i + 2$ where the vertex vi = vi1 is in γt(Pi)-set. Thus γt(G0) = $\sum_{i=1}^{n} \lceil \frac{n_i}{2} \rceil$.

Example: Peterson Graph with minimum Total dominating set St = {4, 8, 9, 10}
It's γt(G0) = 4

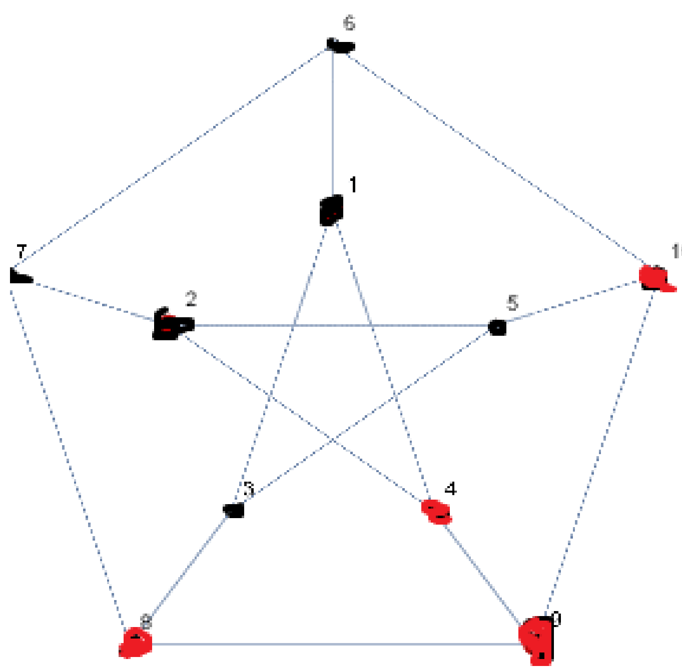

Figure 6. Peterson Graph

Graph G0 obtained by adding a path P6 having 4 vertices at vertex 6 as follows

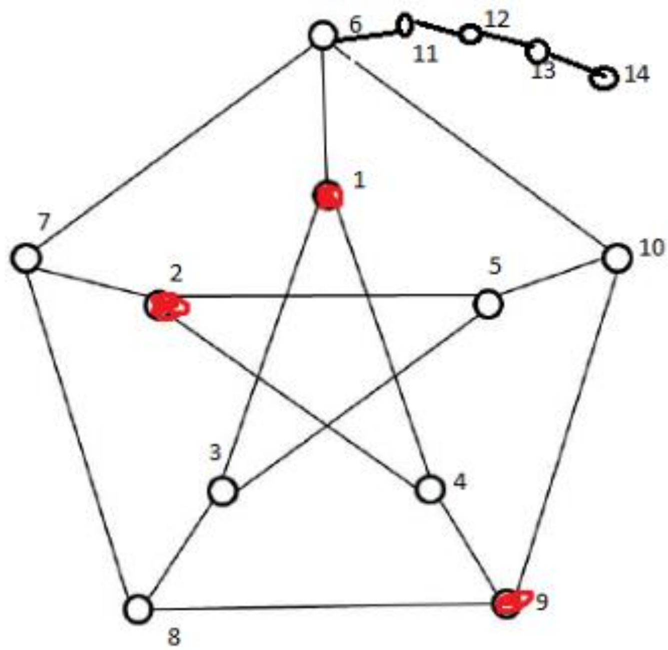

Figure 7. G_0 Graph by adding a path P6

Since n6 = 4 ≠ 4k6+1 for any k6. So **γt**(G0) = $\lceil \frac{n_6}{2} \rceil = \lceil \frac{4}{2} \rceil = 2$.

Note: Above theorems can be used to find feasible solution of dominating sets *S*1, *S*2, ,Sn from given graph for applying to the following algorithms as input to obtain the Minimum (Total)dominating sets.

4. CONSTRUCTING AN OPTIMAL [1, 2] - DOMINATING SET IN A GRID GRAPH

Step 1: Obtaining the dominating vertices of the Subgrid of Grid Graph and consider some vertices of the boundary as dominating points.

a. Let m denotes row and n denotes column of the grid. Obtain the first dominating vertex v_{i,r_1} in row m where r_i can be found from the below

$$r_1 = \begin{cases} 2 & \text{if } 5 \text{ divides } n \\ n \bmod 5 & \text{otherwise} \end{cases}$$

For other rows $r_i = (r_1 + 3(i-1)) \bmod 5 \qquad 2 \le i \le m$

b. Obtain the following sets of dominating vertices

$$F_D = \{v_{1,\ 5k+r_1} \text{ where } 3 \le 5k + r_1 \le n-2 \text{ for some } k\}$$

$M_D = \{v_{i,\ 5k+r_i}\ where\ 2 \le i \le n-1\ and\ 1 \le 5k+r_1 \le n\ for\ some\ k\}$

$L_D = \{v_{i,\ 5k+r_i}\ where\ 3 \le 5k+r_1 \le n-2\ for\ some\ k\}$

Step 2: In this step, we find set of those vertices in the boundary of grid which are not dominated. We add the following vertices to the dominating set.
Let

A $= \lfloor \frac{n}{5} \rfloor$, $B = \lfloor \frac{m}{5} \rfloor$ Define $S_k^{(a,b)} = \{5t+k: a \le t \le b\}$

a. In First Row of the boundary: In this step, we find vertices in first row of boundary those are not dominated yet denote it as the set B_D^{FR}.

$$B_D^{FR} = \begin{cases} \left\{v_{1,r}: r\epsilon S_4^{(1,\ A-1)}\right\} \cup \left\{v_{1,3}\right\} & if\ n \equiv 0\ mod\ 5 \\ \left\{v_{1,r}: r\epsilon S_3^{(1,\ A-2)}\right\} \cup \left\{v_{1,2}, v_{1,n-2}\right\} & if\ n \equiv 1\ mod\ 5 \\ \left\{v_{1,r}: r\epsilon S_4^{(1,\ A-2)}\right\} \cup \left\{v_{1,3}, v_{1,n-2}\right\} & if\ n \equiv 2\ mod\ 5 \\ \left\{v_{1,r}: r\epsilon S_0^{(1,\ A-1)}\right\} \cup \left\{v_{1,n-2}\right\} & if\ n \equiv 3\ mod\ 5 \\ \left\{v_{1,r}: r\epsilon S_1^{(1,\ A-1)}\right\} \cup \left\{v_{1,n-2}\right\} & if\ n \equiv 4\ mod\ 5 \end{cases}$$

b. In Left, Right Columns and Last Row of boundary: In this step, we find set of vertices $B_D^{LC}, B_D^{RC}, B_D^{LR}$ denotes in left column, right column and last row of the boundary respectively those are not dominated yet.

Case (1): n mod 5 = 0

$$B_D^{LC} = \begin{cases} \left\{v_{r,1}: r\epsilon S_2^{(0,\ B-2)}\right\} \cup \left\{v_{m-3,1}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,1}: r\epsilon S_2^{(0,\ B-1)}\right\} & if\ m \equiv 1, 2\ mod\ 5 \\ \left\{v_{r,n}: r\epsilon S_4^{(0,\ B)}\right\} & if\ m \equiv 3\ mod\ 5 \\ \left\{v_{r,1}: r\epsilon S_2^{(0,\ B-1)}\right\} \cup \left\{v_{m-1,1}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$B_D^{RC} = \begin{cases} \left\{v_{r,n}: r\epsilon S_4^{(1,\ B-1)}\right\} \cup \left\{v_{3,n}\right\} & if\ m \equiv 0, 3, 4\ mod\ 5 \\ \left\{v_{r,n}: r\epsilon S_4^{(1,\ B-2)}\right\} \cup \left\{v_{m-1,n}, v_{m-1,n}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{r,n}: r\epsilon S_4^{(1,\ B-2)}\right\} \cup \left\{v_{m-1,n}, v_{m-2,n}\right\} & if\ m \equiv 2\ mod\ 5 \end{cases}$$

$$B_D^{LR} = \begin{cases} \left\{v_{m,r} : r \epsilon S_2^{(0,\ A-2)}\right\} \cup \left\{v_{m,n-2}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_0^{(1,\ A-1)}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_1^{(1,\ A-1)}\right\} & if\ m \equiv 3\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_3^{(1,\ A-2)}\right\} \cup \left\{v_{m,2}, v_{m,n-1}\right\} & if\ m \equiv 2\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_4^{(1,\ A)}\right\} \cup \left\{v_{m,3}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

Case (2): n mod 5 = 1

$$B_D^{LC} = \begin{cases} \left\{v_{r,1} : r \epsilon S_4^{(1,\ B-1)}\right\} \cup \left\{v_{3,1}\right\} & if\ m \equiv 0, 3, 4\ mod\ 5 \\ \left\{v_{r,1} : r \epsilon S_4^{(1,\ B-2)}\right\} \cup \left\{v_{3,1}, v_{m-1,1}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{r,1} : r \epsilon S_4^{(1,\ B-2)}\right\} \cup \left\{v_{3,1}, v_{m-2,1}\right\} & if\ m \equiv 2\ mod\ 5 \end{cases}$$

$$B_D^{RC} = \begin{cases} \left\{v_{r,n} : r \epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-1,n}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,n} : r \epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}\right\} & if\ m \equiv 2, 3\ mod\ 5 \\ \left\{v_{r,n} : r \epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-2,n}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{r,n} : r \epsilon S_3^{(1,\ B-1)}\right\} \cup \left\{v_{2,n}, v_{m-2,n}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$B_D^{LR} = \begin{cases} \left\{v_{m,r} : r \epsilon S_1^{(1,\ A-1)}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_4^{(1,\ A-1)}\right\} \cup \left\{v_{m,3}\ v_{m,n-1}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_2^{(0,\ A-1)}\right\} & if\ m \equiv 2\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_0^{(1,\ A)}\right\} & if\ m \equiv 3\ mod\ 5 \\ \left\{v_{m,r} : r \epsilon S_3^{(1,\ A-2)}\right\} \cup \left\{v_{m,2}, v_{m,n-2}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

Case (3): n mod 5 = 2

$$B_D^{LC} = \begin{cases} \left\{v_{r,1} : r \epsilon S_2^{(0,\ B-2)}\right\} \cup \left\{v_{m-2,1}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,1} : r \epsilon S_2^{(0,\ B-1)}\right\} & if\ m \equiv 1, 2, 3\ mod\ 5 \\ \left\{v_{r,1} : r \epsilon S_2^{(0,\ B-1)}\right\} \cup \left\{v_{m-1,1}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$B_D^{RC} = \begin{cases} \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-1,n}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-2,n}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-1)}\right\} \cup \left\{v_{2,n}\right\} & if\ m \equiv 2,3\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B)}\right\} \cup \left\{v_{2,n}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$B_D^{LR} = \begin{cases} \left\{v_{m,r} : r\epsilon S_1^{(0,\ A-1)}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_4^{(1,\ A-1)}\right\} \cup \left\{v_{m,n-1}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_2^{(1,A-1)}\right\} \cup \left\{v_{m,2}\right\} & if\ m \equiv 2\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_0^{(1,\ A)}\right\} & if\ m \equiv 3\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_3^{(1,\ A-2)}\right\} \cup \left\{v_{m,3}, v_{m,n-2}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

Case (4): n mod 5 = 3

$$B_D^{LC} = \begin{cases} \left\{v_{r,1} : r\epsilon S_0^{(1,\ B-1)}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,1} : r\epsilon S_0^{(1,\ B)}\right\} & if\ m \equiv 1,4\ mod\ 5 \\ \left\{v_{r,1} : r\epsilon S_0^{(1,\ B-1)}\right\} \cup \left\{v_{m-1,1}\right\} & if\ m \equiv 2\ mod\ 5 \\ \left\{v_{r,1} : r\epsilon S_0^{(1,\ B-1)}\right\} \cup \left\{v_{m-2,1}\right\} & if\ m \equiv 3\ mod\ 5 \end{cases}$$

$$B_D^{RC} = \begin{cases} \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-1,n}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-2)}\right\} \cup \left\{v_{2,n}, v_{m-2,n}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B-1)}\right\} \cup \left\{v_{2,n}\right\} & if\ m \equiv 2,3\ mod\ 5 \\ \left\{v_{r,n} : r\epsilon S_3^{(1,\ B)}\right\} \cup \left\{v_{2,n}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$B_D^{LR} = \begin{cases} \left\{v_{m,r} : r\epsilon S_3^{(1,\ A-1)}\right\} \cup \left\{v_{m,2}\right\} & if\ m \equiv 0\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_1^{(1,\ A-1)}\right\} \cup \left\{v_{m,n-1}\right\} & if\ m \equiv 1\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_4^{(1,\ A-1)}\right\} \cup \left\{v_{m,3}\right\} & if\ m \equiv 2\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_2^{(0,\ A)}\right\} & if\ m \equiv 3\ mod\ 5 \\ \left\{v_{m,r} : r\epsilon S_0^{(1,\ A-1)}\right\} \cup \left\{v_{m,n-2}\right\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

Case (5): n mod 5 = 4

$$B_D^{LC} = \begin{cases} \{v_{r,1}: r\epsilon S_3^{(1,\ B-2)}\} \cup \{v_{2,1}, v_{m-1,1}\} & if\ m \equiv 0\ mod\ 5 \\ \{v_{r,1}: r\epsilon S_3^{(1,\ B-2)}\} \cup \{v_{2,1}, v_{m-2,1}\} & if\ m \equiv 1,4\ mod\ 5 \\ \{v_{r,1}: r\epsilon S_3^{(1,\ B-1)}\} \cup \{v_{2,1}\} & if\ m \equiv 2\ mod\ 5 \end{cases}$$

$$B_D^{RC} = \begin{cases} \{v_{r,n}: r\epsilon S_3^{(1,\ B-2)}\} \cup \{v_{2,n}, v_{m-1,n}\} & if\ m \equiv 0\ mod\ 5 \\ \{v_{r,n}: r\epsilon S_3^{(1,\ B-2)}\} \cup \{v_{2,n}, v_{m-2,n}\} & if\ m \equiv 1\ mod\ 5 \\ \{v_{r,n}: r\epsilon S_3^{(1,\ B-1)}\} \cup \{v_{2,n}\} & if\ m \equiv 2,3,4\ mod\ 5 \end{cases}$$

$$B_D^{LR} = \begin{cases} \{v_{m,r}: r\epsilon S_4^{(1,\ A-1)}\} \cup \{v_{m,3}\} & if\ m \equiv 0\ mod\ 5 \\ \{v_{m,r}: r\epsilon S_2^{(1,\ A-1)}\} \cup \{v_{m,n-1}\} & if\ m \equiv 1\ mod\ 5 \\ \{v_{m,r}: r\epsilon S_0^{(1,\ A)}\} & if\ m \equiv 2\ mod\ 5 \\ \{v_{m,r}: r\epsilon S_3^{(1,\ A)}\} \cup \{v_{m,2}\} & if\ m \equiv 3\ mod\ 5 \\ \{v_{m,r}: r\epsilon S_1^{(1,\ A-1)}\} \cup \{v_{m,n-2}\} & if\ m \equiv 4\ mod\ 5 \end{cases}$$

$$D = F_D \cup M_D \cup L_D \cup B_D^{FR} \cup B_D^{LC} \cup B_D^{RC} \cup B_D^{LR}$$

D is an optimal dominating set for $G_{m,n}$.

5. APPLICATION OF DOMINATING SETS IN WIRELESS SENSOR NETWORK:

Algorithm 1. (LOCAL SEARCH):

Algorithm requires a feasible solution which are the dominating sets *S*1, *S*2, ….. , *Sn*. Now, a swap is attempted between the nodes *vi* and *vj* which are two nodes of two different dominating sets *Sk* and *Sl* respectively. If *Sk* and *Sl* remain dominating sets and the sum of the lifetimes of *Sk* and *Sl* increases, then the attempted swap is made permanent. Otherwise, the swap is not made. In this way, the algorithm attempts swaps between every node of every dominating set where the two nodes are from two different subsets. The algorithm continues as long as lifetime improvements can be made by making new swaps.

Input: dominating sets *S*1, *S*2, *S*3,…. *Sn*
while *improvement* **do**
for *any* (*vi*, *vj*) *where vi* ∈*Sk* , *vj* ∈*Sl* **do**
if *the swap* (*Sk* − *vj* ∪ *vj*, *Sl* - *vj* ∪ *vi*) *increases*
total lifetime of the sets Sk and Sl while not
violating dominating sets constraints **then**
make the swap (*Sk* − *vj* ∪ *vj*, *Sl* - *vj* ∪ *vi*)

Algorithm 2. (FIXED DEPTH)

Here instead of making the first swap that meets the two conditions, it lists all such swaps in a swap list. After finding all the swaps, it makes the swap that provides the greatest increase in the lifetime of the network. Next, the algorithm repeats the previous steps. The process stops when no such swaps is found.

Input: dominating sets *S*1, *S*2, *S*3, …*Sn*
while *improvement* **do**
initialize the gain array, *glist*
for *any* (*vi*; *vj*) *where vi* ∈*Sk* , *vj* ∈*Sl* **do**
if *the swap* (*Sk* − *vj* ∪ *vj*, *Sl* - *vj* ∪ *vi*) *increases*
total lifetime of Sk and Sl and not violating
dominating sets constraints **then**
append *gain* from the swap to *glist*
if *glist in not empty* **then**
find *vi*, *Sl* , *vj*, *Sk* for maximum *gain* in *glist*
make the swap (*Sk* − *vj* ∪ *vj*, *Sl* - *vj* ∪ *vi*)

Algorithm 3. (VARIABLE DEPTH SEARCH)

The variable depth algorithm lists all the feasible swaps. Here, few of the listed swaps can decrease the lifetime of the network. However, it makes the swap that provides the maximum increase of the network lifetime and adds this swap to a swap list. This process continues as longs as feasible swaps can be found. If a swap is made between two nodes *vi* and *vj* in any of the steps, the subsequent steps will not have a swap involving the same two nodes. In the next step, the algorithm finds a continuous subsequence from the swap list. This subsequence will always start from the beginning of the swap list and try to maximize the sum of the increase of network lifetime. Now, the algorithm makes all the swaps in the subsequence.

Input: dominating sets *S*1, *S*2, *S*3, ….. *Sn*
initialize swap array(*slist*)and gain array(*glist)*
while *true* **do**
initialize the temporary gain array(*tgains)*
for *any* (*vi*, *vj*) *where vi* ∈*Sk* , *vj* ∈ S_l **do**
if *the swap* (*Sk* - *vj* ∪*vj*, *Sl* - *vj* ∪*vi*) *does not*
violate dominating sets constraints **then**
append *gain* from the swap to *tgains*
if *tgains in not empty* **then**
find *vi*, *Sl* , s*vj*, *Sk* for maximum *gain* in *tgains*
append *gain* from
(*Sk* - *vj* ∪*vj*, *Sl* - *vj* ∪*vi*) to *glist*
make the temporary swap
find *k* which maximizes *g_max*, the sum
of *glist*[1], *glist*[2],…. *glist*[*k*]
if *g_max* > 0 **then**
make the swaps for glist[1],..., glist[k]
else
break.

RUNTIME of Algorithms 1, 2,3:

The time complexity (in the worst case) of all three local search algorithms are $O(|V|^2T)$ where $|V|$ is the total sensors and T is the maximum possible network life time. Complexity of these three algorithms are same, the fixed depth algorithm (algorithm 2) takes more time than the simple local search (algorithm 1) and the variable depth algorithm (algorithm 3) takes the time most in practise due the slow improvement of the lifetime in each step.

6. CONCLUSION

In this paper, we presented a dominating set of a graph G if for every v ∈ V, either v ∈ S or v ∈ N(u) for some vertex u ∈ S where N(u) represents neighbourhood of u. Then we proved minimum cardinality of a dominating set in a graph G is called the domination number of G, and is denoted by γ(G).A subset S ⊆ V in a graph G = (V, E) is a total [1, 2]-set if, for every vertex v ∈ V,1 ≤ |N(v) ∩S| ≤ 2. Finally we proved, The minimum cardinality of a total [1, 2]-set of G is called the total [1, 2]-domination number, denoted by $\gamma_{t[1,2]}$(G).

Biography

Dr. P. Rajakumari has a great fervour towards teaching profession. she has completed his Bachelor degree (2011), Master degree (2014) in mathematics at D.K.M.College for women (Autonomous) Vellore, Also completed Master of Philosophy 2015 in Complex Analysis at D.K.M.College for women (Autonomous) Vellore. She has completed B.Ed at K.K.S. Mani College of Education. She has persued Ph.D in Domination of various parameters for middle, central, total, line and zero divisor graphs at D.K.M.College for women (Autonomous) Vellore, under Thiruvalluvar university in the year of 2020. She has worked 5 years as Assistant Professor in Adhiparasakthi College of Arts and Science (Autonomous) at Kalavai. At present she is working as Assistant professor in Jeppiaar Institute of Technology, Chennai. His discipline\Area of specialization in Graph Theory and Algebra. He has published 12 research articles in national and international journals. Shee has research enthusiams Graph Theory and Topological Spaces.

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