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Simultaneous Impacts of Surface Inclination, Magnetic Field and Internal Heat Generation on Thermal Characteristics of Convective-Radiative Porous Fins

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ABSTRACT

The simultaneous influences of surface inclination, magnetic field and internal heat generation in porous fins with radiative-convective heat transfer is explored using homotopy perturbation method. The efficacy of the method is displayed through the verification of the results with numerical method using Runge-Kutta. Also, significance of surface inclination, magnetic field, internal heat generation and other parameters on the heat management enhancement of thermal systems using the analytical solutions presented by the method are discussed. The graphical representation of the thermal behaviour of the extended surfaces is presented for pictorial discussion. The results illustrate that the augmentations of the surface inclination, magnetic field, conductive-radiative, conductive-convective and porosity cause the extended surface temperature to reduce as a result of increased rate of heat flow via the passive device. Also, an increase in the internal heat generation causes the fin temperature to fall and the rate

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of heat transfer from the fin to decrease. It is hoped that the study will assist in proper thermal analysis of fins for effective thermal managements of engineering systems.

***Keywords*:** Fin surface inclination, Magnetic field, Convective-radiative fin, Thermal behaviour, Internal heat generation, Homotopy perturbation method

1. INTRODUCTION

Fins have widely been used for passive cooling in thermal and electronics systems. Although, solid fins have been applied as passive devices for cooling and thermal control of thermal and electronics equipment, further heat transfer enhancement has been achieved through the use of porous fins. The importance of such fins in various thermal and electronic equipment aroused various studies [1-23]. The use of porous fin for heat transfer augmentation was presented by Kiwan and Al-Nimir [1] while Gong et al. [2] illustrated the applications of porous and solid compound fins for heat sink in micro-channel. Ali et al. [3] demostrated experimentally the impacts of heat sink fin shapes and phase change materials on the thermal cooling of electronics. Saedodin [4] explored numericallly the temperature distribution in a free convection porous fins while Sobamowo et al. [5] adopted Galerkin method of weighted residual to scrutinized the temperature distribution in the natural convection porous fins. Oguntala et al. [6] examined the infleunce of deposited particles on the thermal characteristics of heat sink porous fins with convective and radiative heat transfer. Mosayebidorcheh et al. [7] studied the effects of shapes and thermal properties on the transient response of an internally heated fin. Kim and Mudawar [8] also dissected the shapes effects on heat diffusion through an heat sink while Moradi et al.[9] applied differential transformation method to analyze fins with triangular profiles and temperature-variant thermal conductivity.

Various key parameters in the porous fin thermal models have been used for the improved heat transfer enhancement [2-6]. Some of the past works have focused on the utilizations of the fin geometry as well as the thermo-electro-magnetic properties of fin to achieve the optimized heat transfer augmentation of the porous fin [7-14]. In some of the studies, the properties of the surrounding fluid around the passive device have been used to increase the heat dissipating capacity of the fin [15-17]. Additionally, some authors displayed the efficacy of some new analytical and numerical methods in the thermal analysis of the porous fin [18-23]. Thermal behaviours of porous extended surface are studied in other works [24-31].

In the applications of fin for the heat transfer enhancement, it is established that the thermal conductivities of the materials for fins are temperature-dependent. Therefore, the effects of the temperature-dependent thermal properties on the fin performance have been taken into consideration in previous studies. However, it has been established that when there is small temperature variation exists between the base and the tip of the fin, the thermal conductivity of the fin can be taken constant. Also, it has been shown that the thermal conductivity of palladium is constant at a relatively low temperature (-100 °C – 227 °C).

Therefore, thermal analysis of fins with temperature-invariant thermal conductivity has applications in such situations. In some other studies, the effects of inclination of fins on the thermal performance of the extended surfaces have been studied by Sobamowo et al. [32], Gireesha and Sowmya [33] and Jasim and Söylemez [34], Oguntala et al. [35-38] and Amirkolaei *et al.* [39].

However, to the best of the authors' knowledge, there is no study on the analytical investigations of the simultaneous effects of magnetic field, internal heat generation and surface inclination of the fin on thermal behaviour of porous fin with convective and radiative heat transfer. Therefore, in this work, thermal behaviour of convective-radiative porous fin with surface inclination, magnetic field and internal heat generation using homotopy perturbation method. The results of the approximate analytical method are verified with the results of the numerical method using Runge-Kutta. Effects of various parameters on the thermal behaviour of the porous fin are investigated and discussed.

2. PROBLEM FORMULATION

Consider a rectangular fin with pores having convective and radiative heat transfer as shown in Fig. 1. The fin is inclined at an angle γ to the horizontal axis (x-axis) i.e. at an angle 90-γ to the vertical axis (y- axis) as shown in Fig. 2. In order to derive the thermal model of the porous fin, it is assumed that the porous medium is isotropic, homogeneous and it is saturated with single-phase fluid. The physical and thermal properties of the fin and the surrounding fluid surface are constant. The temperature varies in the fin is only along the length of the fin as shown in the Fig. 1. and there is a perfect contact between the fin base and the prime surface.

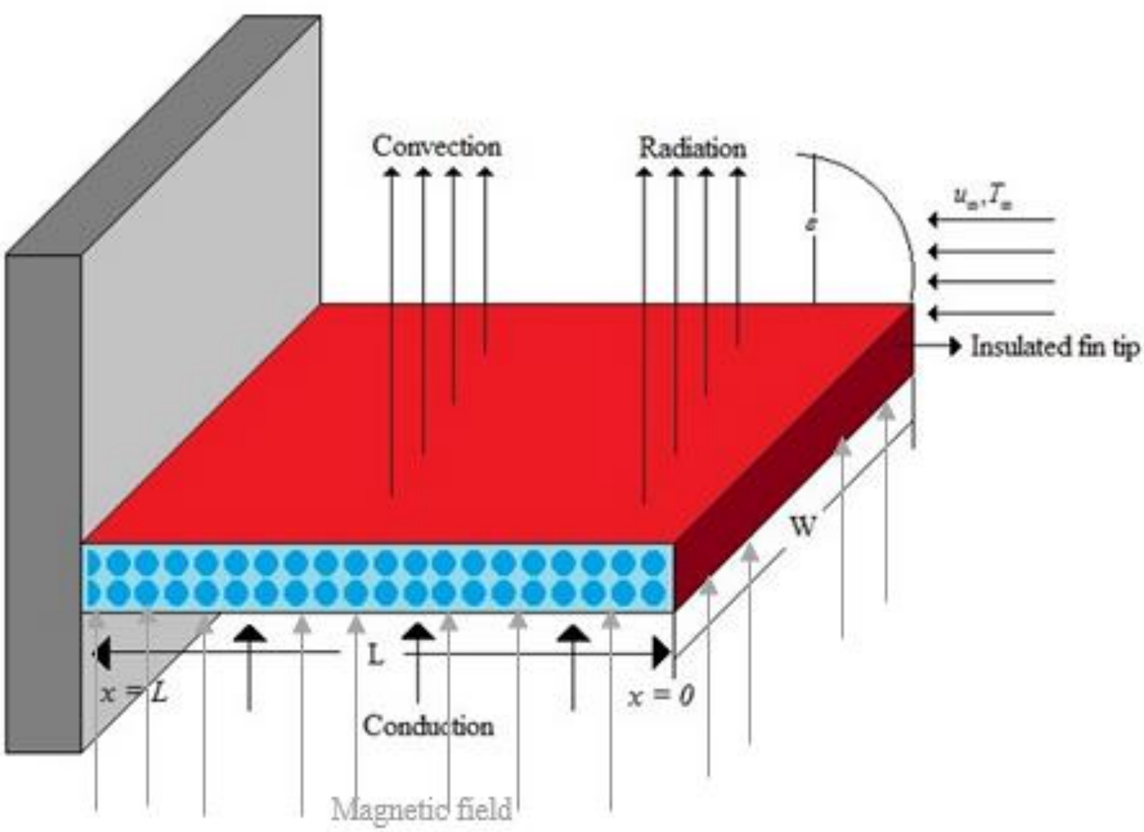

Fig. 1. Convecting-radiating porous fin

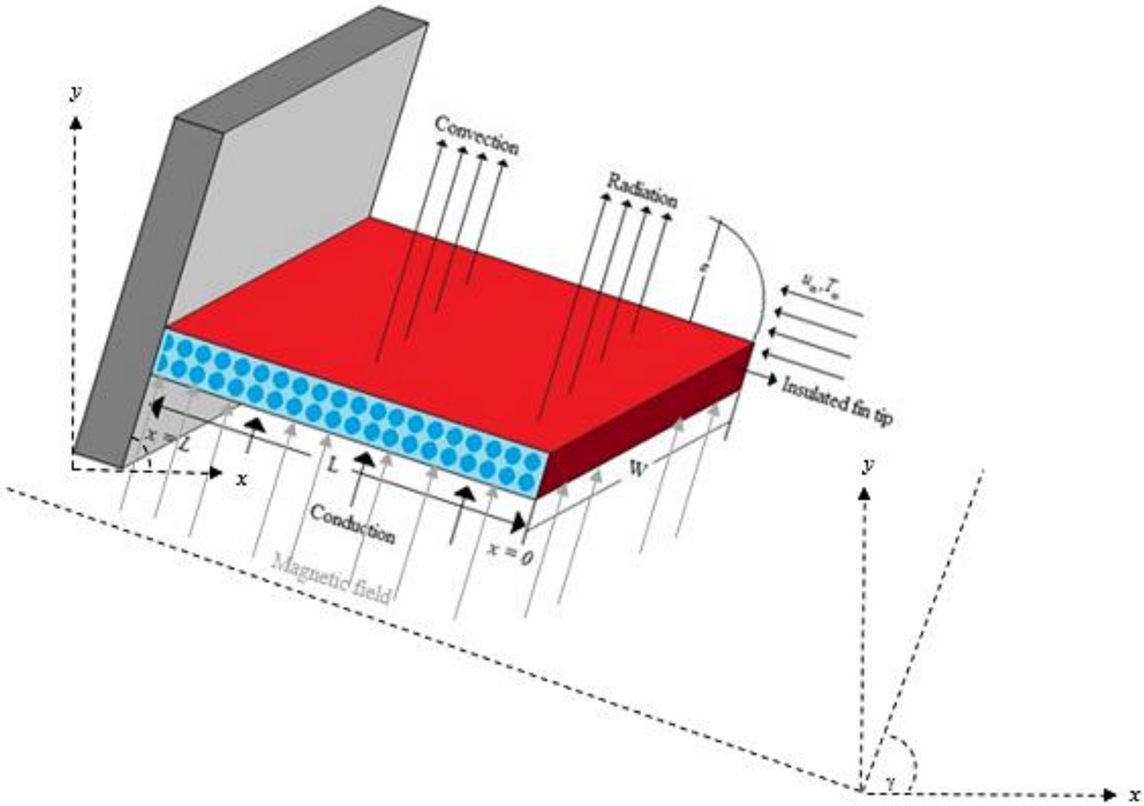

Fig. 2. Convective-radiative porous fin inclined at an angle γ to the horizontal axis (x-axis)

From the assumptions and with the aid of Darcy's model, the energy balance is

$$q_x - \left(q_x + \frac{\delta q}{\delta x} dx \right) + q\left(T\right) dx = \dot{m} c_p (T - T_a) + h(1-\varepsilon)P(T - T_a)dx + \sigma\varepsilon P(T^4 - T_a^4)dx + \frac{J_c \times J_c}{\sigma} dx \tag{1}$$

The fluid flows through the pores at the rate of mass flow given as

$$\dot{m} = \rho u(x) W dx \tag{2}$$

Also, the fluid velocity is given as

$$u(x) = \frac{gK\beta_R \left(T - T_a\right) cosin(90 - \gamma)}{\nu} \tag{3}$$

Using trigonometry identity, the fluid velocity can be written as

$$u(x)=\frac{gK\beta_R\left(T-T_a\right)sin(\gamma)}{\nu} \tag{4}$$

Then, Equ. (1) becomes

$$\begin{aligned}&q_x-\left(q_x+\frac{\delta q}{\delta x}dx\right)+q\left(T\right)dx+\frac{\rho c_p gK\beta_R W\left(T-T_a\right)^2 sin(\gamma)}{\nu}dx\\&+h(1-\varepsilon)P(T-T_a)dx+\sigma\varepsilon P(T^4-T_a^4)dx+\frac{J_c\times J_c}{\sigma}dx\end{aligned} \tag{5}$$

As $dx\rightarrow 0$, Eq. (5) reduces

$$\begin{aligned}&-\frac{dq}{dx}+q\left(T\right)+\frac{\rho c_p gK\beta_R W\left(T-T_a\right)^2 sin(\gamma)}{\nu}\\&+h(1-\varepsilon)P(T-T_a)+\sigma\varepsilon P(T^4-T_a^4)+\frac{J_c\times J_c}{\sigma}\end{aligned} \tag{6}$$

Applying Fourier's law for the heat conduction in the solid, one has

$$q_c=-k_{eff}A_{cr}\frac{dT}{dx} \tag{7}$$

where the effective thermal conductivity of the fin is given as

$$k_{eff}=\phi k_f+(1-\phi)k_s \tag{8}$$

According to Roseland diffusion approximation, the radiative heat transfer rate can be written as

$$q_r=-\frac{4\sigma A_{cr}}{3\beta_R}\frac{dT^4}{dx} \tag{9}$$

From Eqs. (7) and (9), the total rate of heat transfer is given by

$$q_T=-k_{eff}A_{cr}\frac{dT}{dx}-\frac{4\sigma A_{cr}}{3\beta_R}\frac{dT^4}{dx} \tag{10}$$

Substitution of Eq. (10) into Eq. (7) leads to

$$\frac{d}{dx}\left(k_{eff}A_{cr}\frac{dT}{dx}+\frac{4\sigma A_{cr}}{3\beta_R}\frac{dT^4}{dx}\right)+q(T)=\frac{\rho c_p gK\beta(T-T_a)^2 sin(\gamma)}{v}$$
$$+hP(1-\varepsilon)(T-T_a)+\sigma\varepsilon P(T^4-T_a^4)+\frac{\mathbf{J_c}\times\mathbf{J_c}}{\sigma} \tag{11}$$

Expansion of the first term in Eq. (11), provides the governing equation for the required heat transfer

$$\frac{d^2T}{dx^2}+\frac{4\sigma}{3\beta_R k_{eff}}\frac{d}{dx}\left(\frac{dT^4}{dx}\right)-\frac{\rho c_p gK\beta(T-T_a)^2 sin(\gamma)}{k_{eff}tv}-\frac{h(1-\varepsilon)(T-T_a)}{k_{eff}t}$$
$$-\frac{\sigma\varepsilon(T^4-T_a^4)}{k_{eff}t}-\frac{\mathbf{J_c}\times\mathbf{J_c}}{\sigma k_{eff}A_{cr}}+\frac{q(T)}{k_{eff}A_{cr}}=0 \tag{12}$$

The boundary conditions are

$$x=0,\quad \frac{dT}{dx}=0 \tag{13a}$$

$$x=L,\quad T=T_b \tag{13b}$$

But

$$\frac{J_c\times J_c}{\sigma}=\sigma B_o^2u^2 \tag{14}$$

After substitution of Eq. (14) into Eq. (12),

$$\frac{d^2T}{dx^2}+\frac{4\sigma}{3\beta_R k_{eff}}\frac{d}{dx}\left(\frac{dT^4}{dx}\right)-\frac{\rho c_p gK\beta(T-T_a)^2 sin(\gamma)}{k_{eff}tv}-\frac{h(1-\varepsilon)(T-T_a)}{k_{eff}t}$$
$$-\frac{\sigma\varepsilon(T^4-T_a^4)}{k_{eff}t}-\frac{\sigma B_o^2u^2(T-T_a)}{k_{eff}A_{cr}}+\frac{q(T)}{k_{eff}A_{cr}}=0 \tag{15}$$

The term T^4 can be expressed as a linear function of temperature as

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\ldots\cong 4T_\infty^3T-3T_\infty^4 \tag{16}$$

Also, it is given that

$$q(T)=q_0\left[1+\alpha(T-T_\infty)\right] \tag{17}$$

Substitution of Eq. (16) into Eq. (15), results in

$$\frac{d^2T}{dx^2}+\frac{16\sigma}{3\beta_R k_{eff}}\frac{d^2T}{dx^2}-\frac{\rho c_p gK\beta(T-T_a)^2 sin(\gamma)}{k_{eff}tv}-\frac{h(1-\varepsilon)(T-T_a)}{k_{eff}t}-\frac{4\sigma\varepsilon PT_a^3(T-T_a)}{k_{eff}t}$$
$$-\frac{\sigma B_o^2u^2(T-T_a)}{k_{eff}A_{cr}}+\frac{q_o}{k_{eff}A_{cr}}\left[1+\alpha\left(T-T_a\right)\right]=0 \quad (18)$$

Applying the following adimensional parameters in Eq. (19) to Eq. (18),

$$X=\frac{x}{L},\theta=\frac{T-T_a}{T_b-T_a},\ S_h^2=\frac{gk\beta\left(T_b-T_\infty\right)L}{\alpha\nu k_r},\ Nc=\frac{pbh}{A_b k_{eff}},$$
$$Rd=\frac{4\sigma_{st}T_\infty^3}{3\beta_R k_{eff}},\ Nr=\frac{4\sigma_{st}bT_\infty^3}{k_{eff}},\ Ha=\frac{\sigma B_0^2u^2}{k_{eff}A_b}.\ \lambda=\alpha\left(T_b-T_\infty\right) \quad (19)$$

One arrives at the adimensional form of the governing in Eq. (18) as presented in Eq. (20),

$$\frac{d^2\theta}{dX^2}-\frac{S_h sin(\gamma)}{(1+4Rd)}\theta^2-\frac{Nc(1-\varepsilon)}{(1+4Rd)}\theta-\frac{Nr}{(1+4Rd)}\theta-\frac{Ha}{(1+4Rd)}\theta+\frac{M^2Q}{(1+4Rd)}(1+\lambda\theta)=0 \quad (20)$$

and the adimensional boundary conditions

$$X=0,\quad \frac{d\theta}{dX}=0 \quad (21a)$$

$$X=1,\quad \theta=1 \quad (21b)$$

3. METHOD OF SOLUTION BY HOMOTOPY PERTURBATION METHOD

The nonlinearity in the adimensional governing thermal model in Eq. (20) and the boundary conditions in Eq. (21a) and (21b) make it very difficult to generate exact analytical solution for the model. In the work, we adopted homotopy perturbation method. The basic idea of this method can found in our previous work [50]

According to homotopy perturbation method (HPM), one can construct an homotopy for Eq. (20) as:

$$H(\theta,p)=(1-p)[\theta'']+p\left[\begin{matrix}\theta''-\frac{S_h sin(\gamma)}{(1+4Rd)}\theta^2-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\theta\\+\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda\theta)\end{matrix}\right] \quad (22)$$

where $p \in [0,1]$ is an embedding parameter. For $p=0$, and $p=1$, we have

$$\theta(X,0)=\theta_0(X) \quad , \quad \theta(X,1)=\theta_0(X) \tag{23}$$

Note that when p increases from $0 to 1$, $\theta(X,p)$ varies from $\theta_0(X)$ to $\theta_0(X)$. Assuming the that the solution of Eq.(20) can be expressed in a series in p:

$$\theta(X)=\theta_0(X)+p\theta_1(X)+p^2\theta_2(X)+p^3\theta_2(X)+...=\sum_{i=0}^{n}p^i\theta_i(X) \tag{24}$$

Substitution of Eq. (24) into Eq.(22) and expansion of resulting equation, after collecting all the terms with the same order of p together, we have

$$p^0: \quad \theta_0''(X)=0, \quad \theta_0'(0)=0, \quad \theta_0(1)=1 \tag{25}$$

$$\begin{gathered} p^1: \quad \theta_1''+M^2\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_0-\frac{S_h sin(\gamma)}{(1+4Rd)}\theta_0^2-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\theta_0 \\ +\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)=0, \quad \theta_1'(0)=0, \quad \theta_1(1)=0 \end{gathered} \tag{26}$$

$$\begin{gathered} p^2: \quad \theta_2''+\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_1 \\ -\frac{S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_1-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\theta_1=0, \quad \theta_2'(0)=0, \quad \theta_2(1)=0 \end{gathered} \tag{27}$$

$$\begin{gathered} p^3: \quad \theta_3''+\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_2-\frac{S_h sin(\gamma)}{(1+4Rd)}\theta_1^2 \\ -\frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_2-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\theta_1=0, \quad \theta_3'(0)=0, \quad \theta_3(1)=0 \end{gathered} \tag{28}$$

$$\begin{gathered} p^4: \quad \theta_4''-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\theta_3-\frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_2 \\ -\frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_3+\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_3=0, \quad \theta_4'(0)=0, \quad \theta_4(1)=0 \end{gathered} \tag{29}$$

$$p^5: \quad \theta_5'' - \frac{S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_3 + \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_4$$
$$-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\theta_4 - \frac{S_h sin(\gamma)}{(1+4Rd)}\theta_2^{\,2} - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_4 = 0, \quad \theta_5'(0)=0, \quad \theta_5(1)=0 \tag{30}$$

$$p^6: \quad \theta_6'' + \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_5 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_5$$
$$-\frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_4 - \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\theta_5 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_2\theta_3 = 0, \quad \theta_6'(0)=0, \quad \theta_6(1)=0 \tag{31}$$

$$p^7: \quad \theta_7'' + \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_6 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_5 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_6$$
$$-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\theta_6 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_2\theta_4 = 0, \quad \theta_7'(0)=0, \quad \theta_7(1)=0 \tag{32}$$

$$p^8: \quad \theta_8'' + \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_7 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_3\theta_4 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_6$$
$$-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\theta_7 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_7 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_2\theta_5 = 0, \quad \theta_8'(0)=0, \quad \theta_8(1)=0 \tag{33}$$

$$p^9: \quad \theta_9'' - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_0\theta_8 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_2\theta_6 + \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\theta_8 - \frac{S_h sin(\gamma)}{(1+4Rd)}\theta_4^2$$
$$-\frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_3\theta_5 - \frac{2S_h sin(\gamma)}{(1+4Rd)}\theta_1\theta_7 - \left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\theta_8 = 0, \quad \theta_9'(0)=0, \quad \theta_9(1)=0 \tag{34}$$

On solving the above Eqs. (25-34), we arrived at

$$\theta_0(X) = 1 \tag{35}$$

$$\theta_1(X) = \frac{1}{2}\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)} + \frac{Nr}{(1+4Rd)} + \frac{Ha}{(1+4Rd)}\right)\left[1 - \left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right] + \frac{S_h sin(\gamma)}{(1+4Rd)}\right](X^2 - 1) \tag{36}$$

$$\theta_2(X)=\frac{1}{24}\left\{\begin{array}{l}\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right. \\ \left(\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)+\frac{2S_h sin(\gamma)}{(1+4Rd)}\right. \\ \left.-\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\right)^2\end{array}\right\}\left(X^4-6X^2+5\right) \tag{37}$$

$$\theta_3(X)=\left[\begin{array}{l}\frac{1}{8}\frac{S_h sin(\gamma)}{(1+4Rd)}\left(\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right]\right)^2\left(\frac{X^6}{15}-\frac{X^4}{3}+X^2-\frac{11}{15}\right) \\ +\frac{1}{48}\left\{\begin{array}{l}\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right. \\ \left(\left(\frac{Nc(1-\widetilde{\varepsilon)}}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)+\frac{2S_h sin(\gamma)}{(1+4Rd)}\right. \\ \left.-\left(\frac{Nc(1-\widetilde{\varepsilon)}}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\right)^2\end{array}\right\}\left(\frac{X^6}{15}-X^4+5X^2-\frac{61}{15}\right)\end{array}\right] \tag{38}$$

In the similar way, the other terms are found. However, they are too huge to include in this paper.

Following the definition of HPM, it could write that

$$\theta(X)=\theta_0(X)+\theta_1(X)+\theta_2(X)+\theta_3(X)+\theta_4(X)+\theta_5(X)+\theta_6(X)+\theta_7(X)+\theta_8(X)+\theta_9(X)+\ldots \tag{39}$$

On substituting Eqs. (35), (36), (37), and (38), the solution becomes

$$
\begin{aligned}
\theta(X) = 1 &+ \frac{\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]}{2}\left(X^2-1\right) \\
&+\frac{1}{24}\left\{\begin{array}{l}\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right] \\ \left(\begin{array}{l}\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)+\frac{2S_h sin(\gamma)}{(1+4Rd)} \\ -\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\end{array}\right)^2\end{array}\right\}\left(X^4-6X^2+5\right) \\
&+\left[\begin{array}{l}\frac{1}{8}\frac{S_h sin(\gamma)}{(1+4Rd)}\left(\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right]\right)^2\left(\frac{X^6}{15}-\frac{X^4}{3}+X^2-\frac{11}{15}\right) \\ +\frac{1}{48}\left\{\begin{array}{l}\left[\left[\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left[1-\left(\frac{Q}{(1+4Rd)}\right)(1+\lambda)\right]+\frac{S_h sin(\gamma)}{(1+4Rd)}\right]\right] \\ \left(\begin{array}{l}\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)+\frac{2S_h sin(\gamma)}{(1+4Rd)} \\ -\left(\frac{Nc(1-\varepsilon)}{(1+4Rd)}+\frac{Nr}{(1+4Rd)}+\frac{Ha}{(1+4Rd)}\right)\left(\frac{Q}{(1+4Rd)}\right)\lambda\end{array}\right)^2\end{array}\right\}\left(\frac{X^6}{15}-X^4+5X^2-\frac{61}{15}\right)\end{array}\right]+\ldots
\end{aligned}
\tag{40}
$$

4. RESULTS AND DISCUSSION

The approximate analytical solution presented in Eq. (40) was coded and simulated using MATLAB. The efficacy of the homotopy perturbation method to the problem under investigation is displayed through the verification of the results with numerical method using Runge-Kutta method as presented in Table 1. The table shows very good agreement between the results of the HPM and that of the numerical method.

The significance of fin surface inclination, convective-conductive, conductive-radiative, Hartmann number (magnetic field) and porosity numbers on the fin adimensional thermal characteristics are showcased graphically in Figs. 3-7.

Table 1. Comparison of results of NM and HPM for θ(X) for Rd = 0.5, **ε** = 0.1, γ = π, S_h = 0.4, Nc = 0.3, Q = 0, Nr = 0.2, Ha = 0.1

X	**NM**	**HPM**	**Difference**
0.00	0.863499231	0.863499664	0.000000433
0.20	0.868776709	0.868776261	0.000000448
0.40	0.884696967	0.884696500	0.000000467
0.60	0.911531120	0.911530658	0.000000462
0.80	0.949741555	0.949741203	0.000000352
1.00	1.000000000	1.000000000	0.000000000

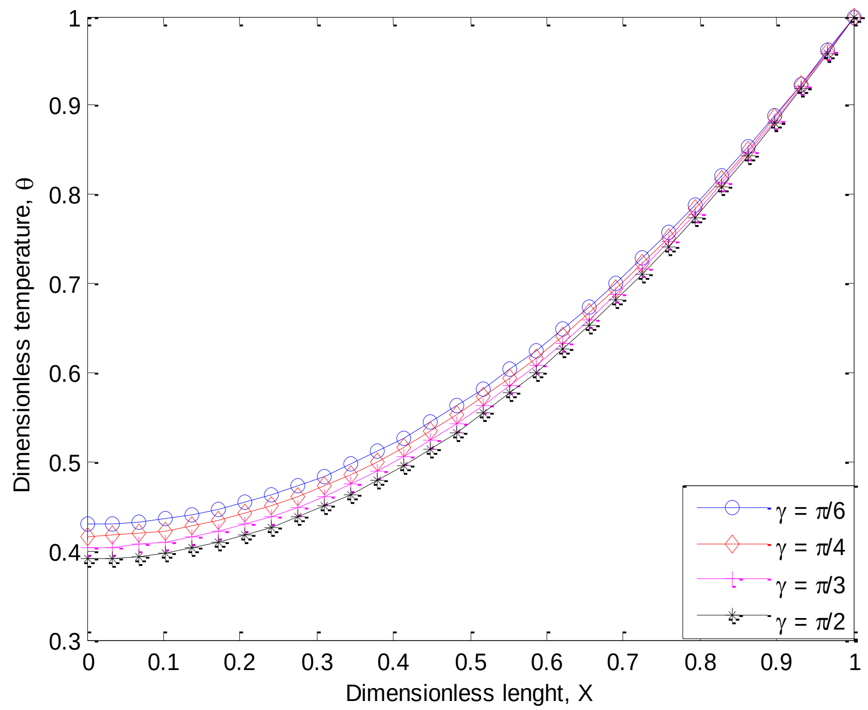

Fig. 3. Impacts of fin surface inclination on fin adimensional thermal distribution

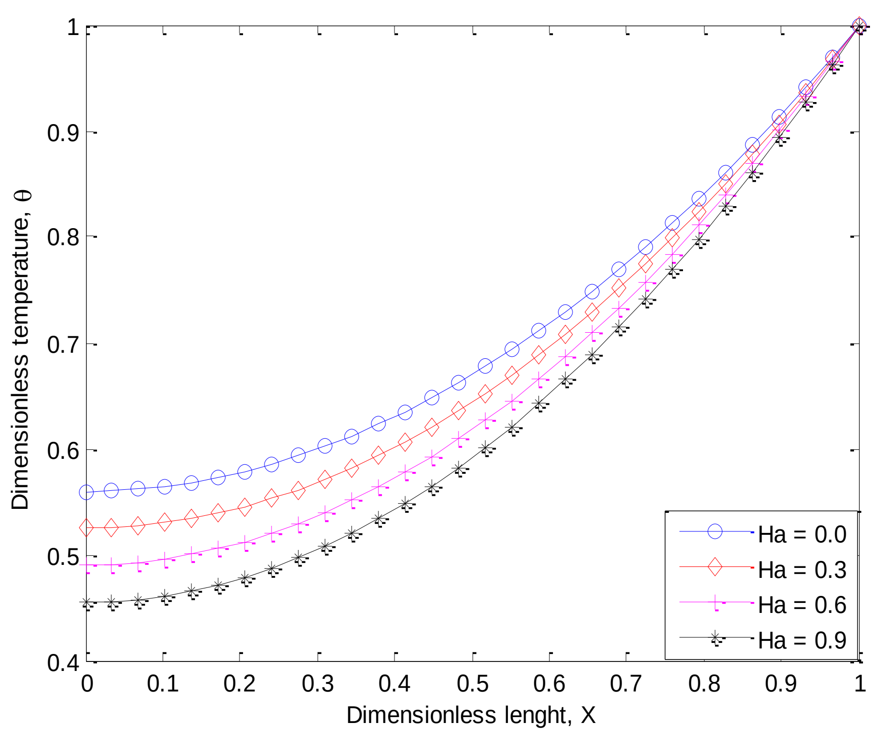

Fig. 4. Impacts of Hartmann number on fin adimensional thermal distribution

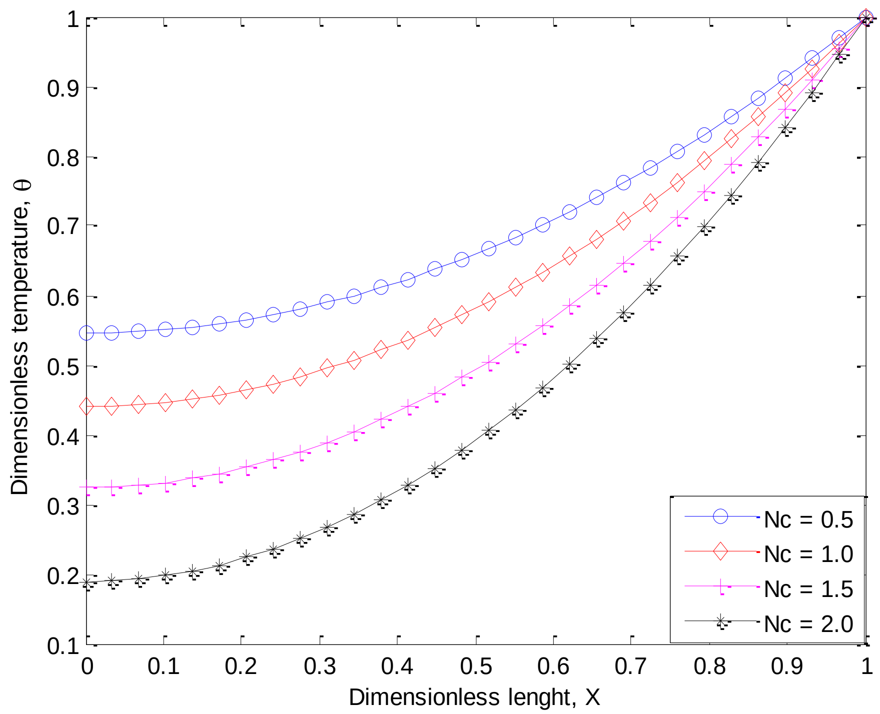

Fig. 5. Impacts of convective number on fin adimensional thermal distribution

The impact of the fin surface inclination on the thermal action of the passive device is illustrated in Figs. 3. When the inclined angle is intensified, the fin adimensional thermal distribution fin reduces. Such declined response in the fin local temperature is due to increased driving force in the buoyancy and convective heat transfer of the working fluid around the passive device.

The Hartmann number represents magnetic field or Lorentz's force influence. Its effect on the fin adimensional temperature is inversely proportional as depicted in Fig. 4. Such response is due to the fact that an increase in the Hartman number lead to increase in Lorentz force which generate resistive force that opposes motion of the fluid surrounding the fin.

Fig. 5, 6 and 7 depict the significances of convective-conductive and conductive-radiative, and porosity numbers on the thermal action of the extended surface. It is illustrated that when the convective-conductive and conductive-radiative, and porosity numbers are augmented, the adimensional fin temperature diminishes which implies high thermal efficiency and effectiveness.

This is due to the fact that the amplifications of the parameters enhance the convective-radiative heat transfer rate from the fin to the surrounding and making the working fluid to increasing infiltrate into the fin pores and increase the buoyancy force effect, thereby causing the effective cooling of the fin.

The effects of internal heat generated on the fin temperature are displayed in Fig. 8 and 9. Since the generated internal heat are additional interior heat to the passive device, their increase cause the fin temperature to rise due to the fact that the conductive heat transfer in the solid body increases as the internal heat is amplified.

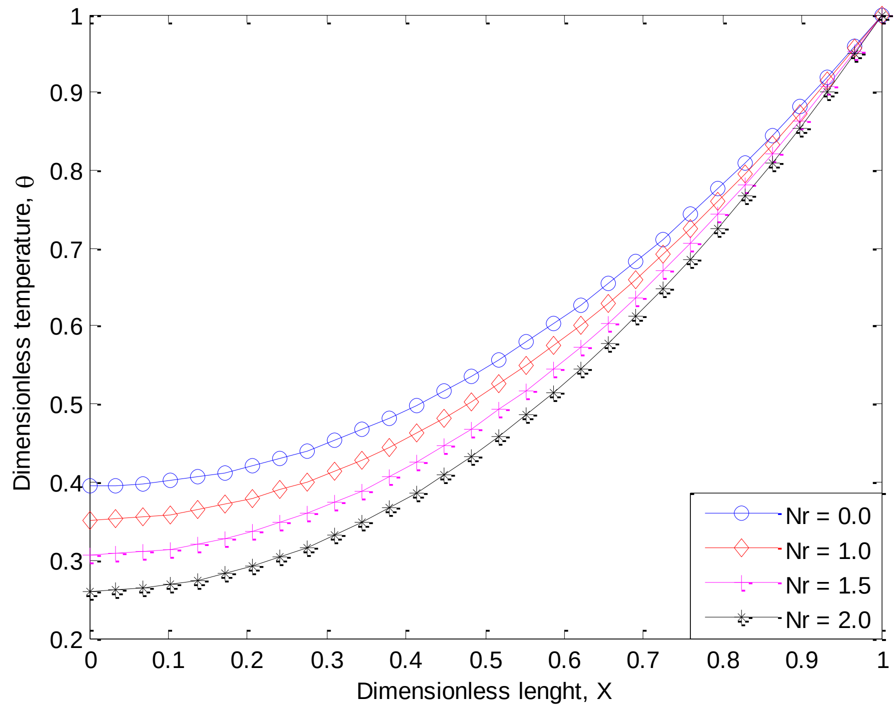

Fig. 6. Impacts of radiative number on fin adimensional thermal distribution

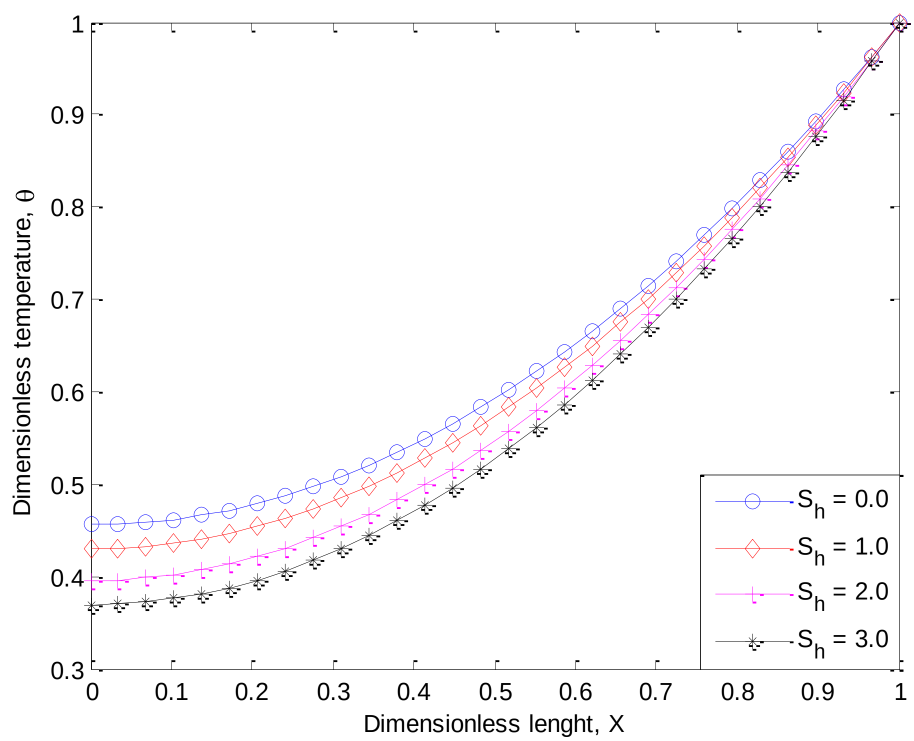

Fig. 7. Impacts of porosity number on fin adimensional thermal distribution

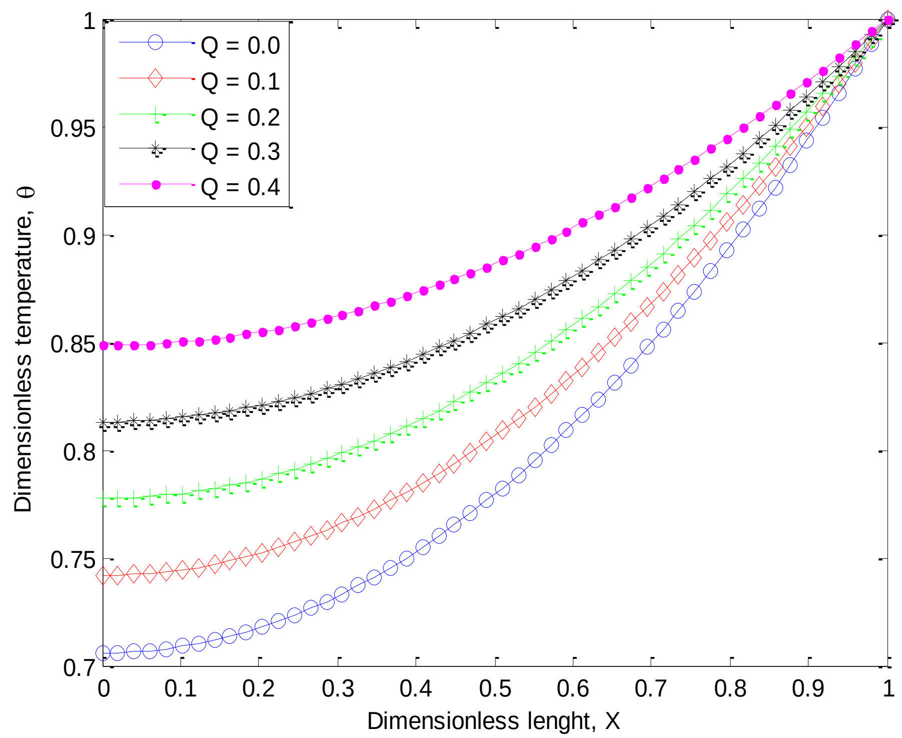

Fig. 8. Impacts of Internal heat generation number on fin adimensional thermal distribution

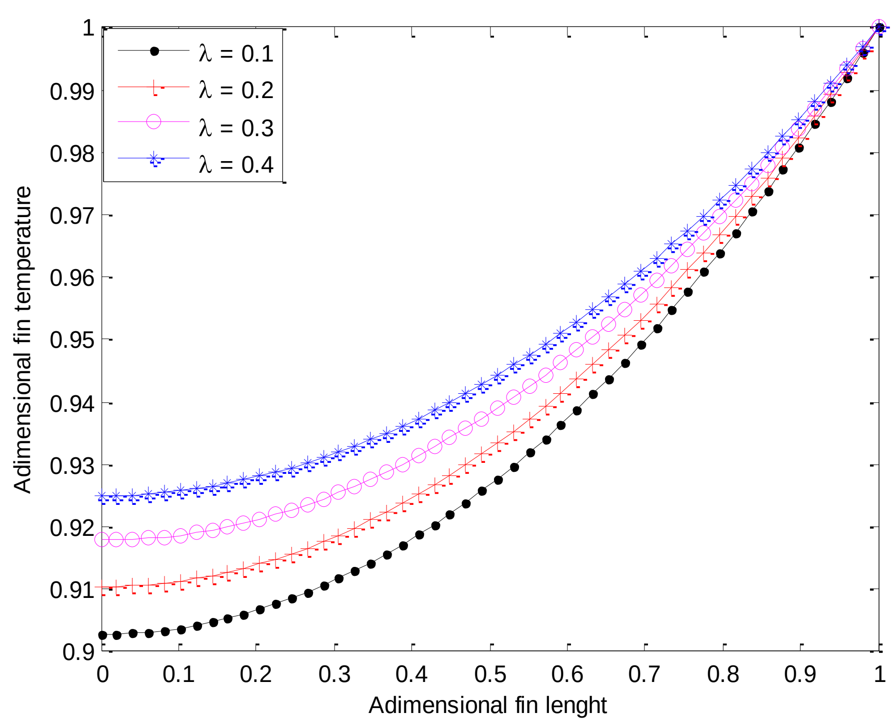

Fig. 9. Effects of variable internal generation parameter on the adimensional temperature distribution in the fin

5. CONCLUSION

In this work, the thermal behaviour of convective-radiative porous fin with surface inclination, magnetic field and internal heat generation have been studied using homotopy perturbation method. The results of the approximate analytical method were verified with the results of the numerical method using Runge-Kutta. The analytical investigation revealed that as the surface inclination, magnetic field, convective, and radiative parameters increase, the adimensional fin temperature decreases which leads to increase in the heat transfer rate through the fin and the thermal efficiency of the fin. Also, an increase in the thermal conductivity and the internal heat generation cause the fin temperature to fall and the rate of heat transfer from the fin to decrease. Therefore, the operational parameters of the fin must be carefully selected to avoid thermal instability in the fin. Also, this study will help in proper thermal analysis of fins and in the design of passive heat enhancement devices used for thermal and electronic systems.

Nomenclature

A cross sectional area,
A_b porous fin base area
c_p specific heat capacity of the fluid passing through porous fin,
h heat transfer coefficient
k_{eff} effective thermal conductivity
L fin length
Rd Radiation number
S_h Porosity number
t fin thickness
T_b fin base temperature
T fin temperature
T_a ambient temperature, K
u average velocity of fluid
x fin axial length
X adimensional length of fin
w fin width

Greek

θ adimensional temperature
ε porosity/void ratio
υ kinematic viscosity
ρ fluid density

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